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# The Information Technologies on Dynamic Chaos for Telecommunication, Radar and Navigation Systems

## Contents

<b>1. Introduction</b>	<b>91</b>
<b>2. The Broadband Signals on the Basis of Chaotic Discrete Algorithms with Nonlinear Dynamics</b>	<b>93</b>
<b>3. The Discrete Generating Algorithms for Forming of Chaotic Signals</b>	<b>93</b>
<b>4. The Structured Complexity of Chaotic Discrete Signals</b>	<b>96</b>
<b>5. The Structured Complexity of Chaotic Binary Sequences</b>	<b>97</b>
<b>6. The Fractal Analysis Methods of Chaotic Algorithms</b>	<b>99</b>
<b>7. The Chaotic Noise-like Signals in Digital Radio Systems with Spread Spectrum</b>	<b>102</b>
<b>8. Conclusion</b>	<b>105</b>

## Abstract

The perspective directions of information technology development on the basis of the dynamic chaos for transmission, processing, storing and protection of information are considered. The discrete chaotic signals with high information capacity on the basis of the nonlinear systems with dynamic chaos were designed and investigated. The analysis of structured complexity and fractal characteristics of pseudorandom integer and binary sequences was carried out. On the models of digital radio channel with spread spectrum it was experimentally shown that designed chaotic algorithms can be effectively used in different radio system, including telecommunication as well as in radiolocation and navigations.

## 1. Introduction

The retrieval of information carriers (the processes and signals) with high information capacity and mathematical algorithms generating such processes is the most actual problem at development of new information technologies. The basic abstract conception in the area of information technologies is "information coding" which interpreted usually as the synonym of conception "information presentation". Such information carriers may be a graphs (drawings), texts, musical notations, numbers, the sequences of electromagnetic, optical or the other signals.

The term "information systems" includes all types of systems providing receiving, processing, transmitting

and storage of information. These are a various types of sensors transforming the external influence (sound, image as light fields of different local intensity, pressure, temperature, chemical composition of media and others) into electrical signals. These are an electronic systems of transformation and processing these signals, computing networks, telecommunication wireless systems. The information in these systems is writing down as continuous signals (an analog form of information encoding) or as sequences of electrical pulses (a digital form of encoding). In case of analog encoding the necessary information is transmitted with help amplitude or frequency manipulation of the continuous electric signal. In case of binary digital signal the logical state "0" is corresponded the absence

of electrical voltage (or current) and for the logical state "1" they are present. The digital codes because of their good error and noise immunity, high rate of processing in computing systems and high density of transmission in communication's channels are the most widely used in the modern telecommunication systems.

In the middle of last century there appeared a great number of publications on discovery of chaotic movements in various systems and mediums. As a result there was started a scenario's classification of transition from monochromatic oscillations to chaos for different dynamical systems [1]. The front expansion of these investigations allowed to clearly recognize the rule of starting a chaotic movement. As a result it was lead to design of "a strange attractor" conception as an image of dynamic system's movement in its phase space. Today the conception of dynamical chaos is identified with the "strange attractor" conception which was introduced into practice by Ruell and Tackens [2]. The specific feature of dynamical system's movement on the strange attractor is the fact that unstable trajectory in its phase space is attracted not to the limit cycle or torus with integer number dimension but to the stable Cantor's manifold with the fractal type dimension [3]. Earlier the complex dynamic of conservative systems was discussed in publications [4].

The subsequent investigations were carried out on the way of finding the more wider classes of dynamical systems (at that as the physical systems representing natural phenomenon and also as the mathematical models and technical systems) for which there were observed the effects of chaotic type. There were determined the rules of scenarios transition to chaos and cleared up the necessary conditions for existing of chaotic type's movement. The chaotic movement of dynamic systems has a number of specific features. The realizations of this movement have a continuous power spectrum in wide region of frequency, an exponentially decay of autocorrelation function, a Gauss' type function of probability distribution. At the same time for the systems with dynamical chaos there are such indeed dynamical features as extremely high sensitivity to the starting conditions and connected with it an exponential scattering of originally closed trajectories. The actual direction of the investigation is the development of new technologies on the basis of specific properties of systems with dynamic chaos. It becomes a ground for development series of new directions for investigations including in such areas as informatics, biophysics, chemistry, medicine, dynamic of natural phenomenon (for example, earthquake) [5].

The application of information technologies presumes the physical realization of the concrete encoding process at transmission, processing and storing of information in telecommunication systems

and computer networks. In addition the information about polynomials of high degree suitable to noise proof coding is locked for free access. In the first place this is dictated by necessity the development of efficient information channels and reliability of the control distributed networks and automatic systems with remote control in which the price of error or partial loss of information can have a disastrous consequences up to loss of the whole system.

As result of existing at the last years tendency to global spreading of the varied opened telecommunication systems and escalated of abonents growing there is the necessity of the information protection not only for special services or business but also for practically each individual user. In telecommunication systems this problem is connected with not only  $h$  the information confidentiality as with loss of information because of low noise-immunity of various information channels. Especially significant the problem of increasing of noise-immunity is for wireless systems. The saturation of the frequency range by traditional communication systems founded on principle of frequency division channels has conducted to development methods of code division channels in which as codes the stream of pseudorandom numbers is used. At information transmission the stream coding provides most noise-immunity (and consequently cryptoresistance) at using of the continual pseudorandom key flows with uniform distribution function.

One of such perspective direction is investigation in area of formation new telecommunication systems on the basis of chaotic dynamic's properties. The systems with chaotic dynamic give ability to get the complex oscillations in devices with simple structure; the realization in simple system of a large number of different chaotic modes; a large information capability; the variety entering methods of information signal into chaotic carrier; the synchronization capability of transmitter and receiver; the confidentiality at transmitting and others. Such type diversity characteristics of chaotic dynamical systems was the reason for different approach to use chaotic modes of dynamical systems in information technologies. The important direction of investigations is development of new classes of algorithms for forming sequences with properties of random processes on the basis of chaotic dynamics [6,7]. There are open possibility to develop a new information technologies and to produce a new perspective methods of complex chaotic signals application in telecommunication systems and also in systems of radar and navigation for transmitting, processing, storing and protection of information.

The radar objects recognition at small signal to noise ratio is one of the most complex problem for modern radar. The information using for object recognition is contented in structure of reflected radar signals. The most perspective route for solution a

problem of radar objects recognition is application of neurocomputing algorithms for signal processing.

The multifunction and the efficiency of modern radar systems may be provided only by development and application of nontraditional digital algorithms and a new adaptive applied solution on processing signals and images for separating and recognizing of objects with small contrast.

At present time in connection with development digital radars and wide implantation of wideband probing signals the interest to this problem substantially grown up [8]. The increasing of precision and resolution of radar measurements is connected with structure's complication and expanding frequency band of probing pulse. This expanding may be to get by using frequency or phase modulation of continuous or quasi-continuous carrier. The extreme case of continuous or quasi-continuous probing signal is so called the white noise with a uniform frequency spectrum other words this signal has  $\delta$ -type uncertainty function. Such type signal provides high precision single meaning measurements as of distance to object and so of radial component speed of object. An added bonus of continuous wideband noise is possibility of providing good signal to noise ratio at the input of receiving device for comparison with pulse signals. In case of using pulses with supershort duration for providing satisfactory signal to noise ratio it is necessary to have a huge level of signal's power in pulse while at continuous mode the required value of signal to noise ratio is easy achieved at power's level on many order of magnitude less then power in pulse mode.

## 2. The Broadband Signals on the Basis of Chaotic Discrete Algorithms with Nonlinear Dynamics

The large information capacity and the high structure complexity broadband signals enables to use the fractal adaptive methods of the such signal's processing in coherent and noncoherent radiolocation [9]. Such signals can be produced in the form of the pseudorandom impulse sequences which possess specified spectral and correlation features. Now in the systems with spectrum's expansion there are used pseudorandom sequences (PRS) with maximal period which properties were studied well. These so called M-sequences are forming by simple algorithms and are using successfully in implementations for variety of purposes during more than 40 years. The well-known methods of forming pseudorandom sequences have certain defects and not always allows to satisfy in full measure to requirements of the great signal's system. So in the maximum period sequences family

on the basis of Adamara's function there is a great probability of the appearance of segment pair PRS with high level of the mutual correlation and big amount (up to one third of the PRS length) coinciding symbols [10].

At present the most perspective method of generating PRS is the application of chaotic algorithms describing the complex nonequilibrium behavior of the nonlinear dynamic systems. For using in radio-engineering systems it was proposed a new class of the random sequences formed on the basis of algorithms describing behavior of self-oscillation systems with delay having modes of the dynamic chaos [6]. The particularity of such systems is their nonlinearity and nonperiodical temporary process generated by them. By changing parameters of such dynamical systems and its initial conditions it is possible to vary over a wide range the character of its behavior and by this means to appropriate a control of the type and characteristics of generated chaotic signals.

The proposed algorithms of forming the chaotic signals are the prototypes of the self-oscillating systems with delayed feedback and strong amplitude-phase nonlinearity. At circulations of the signal in circuit with delayed feedback the system's nonlinearity brings to spectrum expansion of the signal. The width of this spectrum is limited by a filtering characteristics of self-oscillating systems. The correlation between these two competitive factors- increasing spectrum nonlinearity and narrowing spectrum filtering allows to form the chaotic signal with a specified width of the spectrum. The formed signals relate to class broadband chaotic signals.

## 3. The Discrete Generating Algorithms for Forming of Chaotic Signals

From information theory it is known that the stochastic signals generated by random process have the most information capacity. The main development problem at forming of the information carriers in digital telecommunication channels is contained in difficulty of generating random binary sequences with using a short defined key. The requirements to characteristics of the sequences pseudorandom numbers are depend on its concrete applications and as a rule one algorithm unable to satisfy to all of this requirements. The mathematical algorithms on the basis of the key form the pseudorandom numeric sequences (PRS) which must have a raw of necessary characteristics:

**the high quality** — the statistical criterions PRS must be close to random process and have a arbitrary large period;

**the efficiency** — algorithm must be fast and occupy a possible smaller volume of memory;

**the repeatability** — at exact reproducing of the algorithm's initial states the same PRS for realization of any duration must be formed and the small modifications of initial procedure must result in generations of the qualitative different sequences;

**the simplicity** — a formula of the algorithm must be simple for realizing and using.

All this accentuates the urgency of new deterministic algorithm's development providing the production a flow of pseudorandom numbers and satisfying to different systems of requirements.

In spite of the fact that it is known a few algorithms for generating PRS, in practice for generating of binary PRS as a rule there is used a recurrence algorithm. In this case on the grounds of linear recurrence correlation and some initial values it is formed an endless sequence in which any next member of sequence is defined on the basis of previous member. The binary sequences formed on the basis of recurrence correlations are easy to realize on computer and as circuit simulation on basis of high-speed multi-digital binary shift registers.

The attempts to adapt for digital algorithms the operations on real numbers were ended in failure as for dynamic chaos systems any substitution of the real number by its nearest integer-valued number leads to an important changing of the statistics of the resulting sequence. The rounding operation of number contributes the unpredictable perturbation in generating algorithm and a formed by it sequence ceases to be statistically independent and consequently random.

At present the main method of the getting binary PRS is a forming  $M$ -sequences (the sequences with maximum period) on the basis of shift register. A present numerical value of member is defined by linear correlations with some weight (the code) relative to previous member of the sequences. At that the weighting factors are selected to provide the quick decline of correlation function to a values of the order  $1/\sqrt{N}$ , here  $N$  — length of the period  $M$ -sequence. The largest defect of this method is an absence of the mathematical tool allowing to get the algebraic polynomials generating  $M$ -sequences with arbitrary large degree. In addition an information on multinomial of high degree suitable to noiseproof coding is locked for free access.

It is known that a lot of sequences such as linear PRS ( $M$ -sequences, Adamar-sequences, Gold-sequences, Kasamy-sequences and others) and such as nonlinear PRS (the Legendre's sequences, bent-sequences and others) have a certain defects and do not satisfy to some of listed above requirements. The

alternative decision of this problem is enabled by using an noise like signals (NLS) formed with some nonlinear system with dynamic chaos. Such NLS have a correlation characteristics which are not worse than  $M$ -sequences and have practically unlimited set of the period's lengths. This NLS can form the ensembles as binary so and multilayered signal with large volumes. The generating algorithm has nonlinear structure this obstructs its recognition in purposes of the following reproduction at unauthorized access to coded information.

All known chaotic dynamic systems with the small number degrees of the freedom (Lorentz' attractor, Rossler's attractor, Chua's systems, ring systems with delay and amplitude nonlinearity) also do not provide the correlation function with necessary parameters.

The dynamic systems in which simultaneously there are present an dissipative (amplitude) and reactive (phase) nonlinearity have the good statistical characteristic. In self oscillating systems with phase nonlinearity and delay as result of existence of phase nonlinearity there are broken conditions of the phase's balance, condition of the mode's synchronizing. In process of chaotization there are the weakening an interspectrum relationships and more quick (in contrast with other autostochastic systems) splitting correlation in generated signal. The signals with good correlation characteristics can be get in class of the nonlinear recirculating systems with delay. In this systems there are simultaneously present an active (amplitude) and an reactive (phase) nonlinearities. The such system's scheme it is possible to represent in the type of three blocks cycle (Fig. 1).

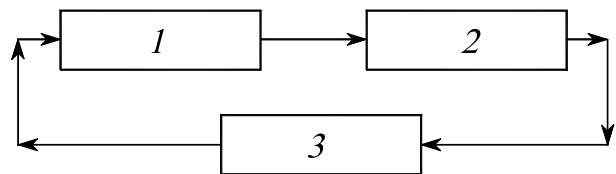


Fig. 1. The model of the dynamic system with chaotic behaviour: 1 — nonlinear amplifier; 2 — line of the delay; 3 — filter.

The self-oscillating mechanism in such system it is possible to describe by the complex integral equation here it is taken into account the interaction of all functional blocks:

$$\hat{x}(t) = \int_{-\infty}^t g(t - \tau) \hat{F}(\tau - T) d\tau. \quad (1)$$

It is possible convert this equation to discrete type:

$$\hat{x}_k = (1 - \exp(-h)) \cdot \hat{F}_{k-N_z} + \exp(-h) \cdot \hat{x}_{k-1}, \quad (2)$$

if enter square-wave filtering the signal, represent the functions  $g$  and as the orthogonal series of Kotelnikov and realize some transformations.

Here  $\hat{x} = a \cdot \exp(i\varphi)$ ,  $\hat{F}_k = F(a_k) \cdot \exp(\varphi + \Phi(a_k))$ :  $N_z$  – delay parameter,  $h$  – step of discretization which chosen in accordance with Kotelnikov’s theorem [11]. The nonlinear transformation functions of the amplitude  $F(x)$  and phases  $\Phi(x)$  of the signal which define the stochastic process in given dynamic system can be enough complex. By the numerical computer analysis there was conducted the system’s parameters choice to achieve the developed chaotic nature self-oscillations and quick falling of auto-correlation functions of the signal (ACF).

The calculation of values of cross-correlation function (CCF) have shown that the form of CCF qualitative similar to ACF but its most peaks in distribution tail area tend to reduction at increasing of duration of sequence realization on similar law  $\alpha/\sqrt{N}$ .

There are two possibilities of the binary signal generation at practical realization of the new class signal in digital technology which is founded mainly on binary code. The first way is connected with clipping of multilayered quantized signals got as a result calculation. As the numerical experiment shown the binary quantization of the multilayered signal practically does not worsen its correlative characteristics.

The second way is the direct construction of discrete self-oscillating systems. For example the generation algorithm of the binary signal in discrete self-oscillating system can be of the form:

$$x_k = \text{sign}[F(x_{k-N})] + x_{k-1}, \quad (3)$$

This correlation is got directly from equation (2).

On the basis of the mathematical model of ring self-oscillating systems with the strong amplitude-phase nonlinearity, the filtration and the delay the discrete generative algorithm of the chaotic signal concerned to class of recurrence-parametric type with delay is designed and investigated. The form of the algorithm of this class in general type is the discrete functional transformation (maps):

$$x_n = f(x_{n-1}, x_{n-2}, \dots, x_{n-N_z}), \quad (4)$$

where  $x_n$  – calculated member of formed pseudorandom sequence on  $n$  step,  $N_z$  – parameter of delay defining number of the sequence members on the delay interval  $x_{n-1}, x_{n-2}, \dots, x_{n-N_z}$  which completely define the new value  $x_n$  and must be given as initial condition on the first step. The function  $f(x)$  transforms the amplitude and phases of signal in the generating ring self-oscillating system in chaos mode. The algorithm is defined on  $M$  ensemble of integer natural numbers belonging to closed numeric interval  $[M_1, M_2]$ , ( $M_2 > M_1$ ,  $M = M_2 - M_1 + 1$ ). This algorithm forms practically uncorrelated pseudorandom sequence of integer numbers with probability distribution close to uniform.

The particularity of investigated algorithms is that its maps formula can remove the new value  $x_n$

for definitional domain of the generating algorithm  $[M_1, M_2]$ . So evident type of the algorithm (4) must be complemented by special operation providing return in given numeric interval  $x_n$  each of following sequence member if it is outside of interval. The similar sort transformations with mapping of numeric ensemble "in itself" known long ago. The example can serve well known "baker transformation" [3]. It is possible and the other types of the transformations but amongst they follow specifically to select that which do not contribute the essential changes to probability distribution of generated numbers.

The phase space (PS) of dynamic system with delay is  $n$ -dimensional space, where  $n$ -number of parameteres uniquely defining behavior of the system on each next step. For system with delay the dimensionality of phase space is defined by number dynamic variables and duration of the delay in feedback presented in discrete type.

The special role among algorithms for forming the random sequences occupies the algorithms for forming the integer sequences. Usually they are defined on limited ensemble of integer numbers that is connected with digit capacity used for presentation integer numbers in digital technology. The advantage of integer number’s sequences is defined by the fact that they are identically reproduced on different types computing devices and at hardware realization are easy to reproduce with circuit technique.

The capacity of used integer number’s ensemble has far less capacity of the permanent continuum of real numbers on which is determined the dynamic system. As a result of this there is a restriction in process of the algorithmic forming of such sequences when by increasing the number of its members there is inevitable output onto cycle which is analogue of the limiting cycle of the dynamic systems determined on permanent number’s continuum. So it is important that on interval before output onto a period of repetitions corresponding to this cycle the realized algorithmic sequences had a statistical characteristics close to characteristics of ideal random sequences.

The used algorithm with delay has a such property that for unambiguous generation whole following sequence it is necessary to specify all  $N_z$  values on interval of delay. It follows that if in the sequence formed by algorithm two non-overlapping segments with the length  $N_z$  are coincide completely with  $L$  step of the algorithm calculation between start points of these segments ( $L > N_z$ ) that sequence will be periodic with period  $T = L$ . The event probability for algorithm specified on integer-valued interval  $[1, 256]$ , has an order of inverse value of the phase space’s volume  $P(256, n) \sim 1/(256)^n = 3 \cdot 10^{-39}$  at  $n = N_z = 16$ .

It is possible to interpret the obtained result as an estimation of the possible period of sequence formed by algorithm. Thereby value of the last one can be  $T \approx$

$10^{38}$  (at  $N_z = 16$ ) of the sequences' members. This estimation it is possible to consider as probable value of the period in formed sequences at parameteres  $M = 256$  and  $N_z = 16$ . It follows that with increasing delay parameter  $N_z$  the probability of the appearance of the period in the sequence formed by algorithm can be made a negligible quality. The computer analysis shows that at arbitrary parameter's values  $M$  and  $N_z$  it is practically always possible to find the long cycle since period  $T = (0.3 \div 1.0)M^{N_z}$ .

The considered algorithm of the chaotic signal forms the multilevel integer signal  $\{x_n\} \in [1, 256]$ . Also there is a common practice of using the systems of binary signals. Such signals it is possible to get from multilevel by using a processes of clipping.

The most complete information about statistical characteristics of the discrete sequences it is given by the analysis of the probability distribution of numbers and conditional probability distribution  $p(i+j, x_n/i, x_k)$ ,  $j = 1, 2, 3, \dots, N$ ,  $n, k = 1, 2, 3, \dots, M$ , i.e. the probability of generations the number  $x_n$  on  $(i+j)$ -step of the algorithm if on  $i$  that step was obtained number  $x_k$ . At definitional domain of the discrete algorithm is arbitrary closed integer-valued interval  $[M_1, M_2]$ ,  $M = M_2 - M_1 + 1$ ,  $x_n \in [M_1, M_2]$ .

If the conditional probability distributions at any  $j$  are practically agreed with uniform distribution so it is follows that all probability of the transition  $p(i+j, x_n/i, x_k) \approx 1/M$ ,  $j = 1, 2, 3, \dots$  at random selection  $i$ . In the same time if probability distribution of generated numbers  $p(x)$  close to even one that probability value of  $x_n$  practically also is equal to  $1/M$ . In that way probability of the transition into state  $x_n$  on  $j$  step is in close agreement with probability of this value on this step regardless of the sequence value on previous step of the algorithm that is typical for ideal random sequences with independent successive samples. Moreover the formed such algorithm the pseudorandom sequences on its probabilistic feature will be close to the sequences of arbitrary equiprobable numbers from interval  $[M_1, M_2]$ . In the last case it is possible to expect that the given sequences will possess the best statistical characteristics. The qualification of the similar fact emphasizes importance of the distribution study of conditional probability for a priori judgment about quality of forming pseudorandom sequences.

For estimation of the conditional probability distribution  $p(x_{i+j}/x_i)$  the big importance has the points locations  $(x_{i+j}, x_i)$  on plane for map  $x_{i+j} = func(x_i)$ , assigned by discrete algorithm at corresponding values  $j = 1, 2, 3, \dots$  and  $i = 1, 2, 3, \dots, N$ . Getting of points scattering  $(x_{i+j}, x_i)$  and visualization on the display does not require the big computing resource in contrast with direct calculation of conditional probability. The nature of this scattering does not give directly form of the distribution of conditional probability however

visualization of the scattering is indicated the degree of regularity of these distribution, presence of the functional relationships, existence of forbidden transitions and forbidden zones.

There was shown that at corresponding choice of parameters the discrete algorithms with delay generate the long nonperiodic segments of pseudorandom sequences with uniform distribution of probability which on statistical and correlation parameters close to features of the random process with even probability distribution function [12].

## 4. The Structured Complexity of Chaotic Discrete Signals

For efficient application of chaotic signals in radio-engineering complex, the telecommunication systems as well as for using them as information carrier in new generation's information technologies it is necessary to develop the methods of the estimation of a structured complexity.

For this purpose there were analyzed the most simple algorithms for forming the pseudorandom sequences of integer-valued numbers  $\{x_n\}$  with delay by using Fibonacci map and its modification:

$$\text{Algorithm F-1 } \tilde{x}_n = x_{n-1} + (-1)^{x_n - K_z} x_{n-N_z}, \quad (5)$$

$$\text{Algorithm F-2 } \tilde{x}_n = x_{n-1} + (-1)^{x_n - N_z} x_{n-N_z}, \quad (6)$$

$$\text{Algorithm F-3 } \tilde{x}_n = x_{n-1} + x_{n-N_z}, \quad (7)$$

where  $N_z$  and  $K_z$  – parameters of algorithms,  $2 \leq K_z \leq (N_z - 1)$ . Unlike work [13] the sign before the delay member in F-1, F-2 changes not at random but one is defined by intrinsic system's dynamics. The feedback parameter  $N_z$  defines the phase space dimension of algorithm and dimension of radius-vector  $R_n(x_{n-1}, x_{n-2}, \dots, x_{n-N_z})$  of the discrete dynamic system state on each step.

The phase space (PS) of the Fibonacci-map with dimension  $N_z$  is not limited. For practical application of PRS algorithms in radio-engineering systems for forming and modulating digital signals with a limited digit capacity it is necessary to assign the definite range of the algorithm on limited numeric set of closed interval of the natural sequence numbers  $[1, M]$  where  $M > 1$ . For these the maps (5)–(7) must be complemented by processing of transformation the numeric interval  $[1, M]$  in itself, for instance, of the following type:

$$\begin{aligned} x_n &= \tilde{x}_n, & \text{if } \tilde{x}_n \in [1, M], \\ x_n &= \tilde{x}_n - M, & \text{if } \tilde{x}_n > M, \\ x_n &= \tilde{x}_n + M, & \text{if } \tilde{x}_n < 1. \end{aligned} \quad (8)$$

This transformation corresponding to rolling up of the length  $[1, M]$  in ring plays the important role

in mechanism of the chaotic behavior of the given dynamic systems. Firstly this operation limits the phase space's volume making its limited and equal to  $V_{PS} = M^{N_z}$  points of the states but secondly this provides additional mixing of system's paths in its phase space. It is necessary to note that only transformations of the numeric interval in to itself it is not enough for efficient mixing of paths in phase space. The certain mechanism of chaotisation must already be embedded in functions of the map. In this case this is provided by features of the Fibonacci-map. These two conditions: the limitation of the volume phase space and the presence of the powerful mechanism of mixing are a necessary conditions of the chaotic behavior of any dynamic systems.

As alternatives there was also considered algorithm (F-4) on the basis of Fibonacci-map (7) but with the other transforming operation of the numeric interval  $[1, M]$  in to itself by type "reflecting borders":

Algorithm F-4

$$\begin{aligned} x_n &= \tilde{x}_n, & \text{if } \tilde{x}_n \in [1, M], \\ x_n &= M, & \text{if } \tilde{x}_n = 2 \cdot M, \\ x_n &= 2 \cdot M - \tilde{x}_n, & \text{if } M < \tilde{x}_n < 2 \cdot M. \end{aligned} \quad (9)$$

In depending on choice of the initial conditions the radius-vector  $R_n$  describes in phase space of the algorithm the path of the consequent discrete transitions from one point of the state of the dynamic system (DS) to another one on the random law. These "paths" of discrete DS in PS because of limitation of the PS volume form the closed cycles which as a result of unambiguaty of the mapping are not crossed and have not a common points. Besides in PS there can be the basins of the cycles and the insulated points. So for instance at  $N_z = 4$ ,  $K_z = 3$  and  $M = 5$  PS of algorithm F-1 has three single cycles with periods 526, 27 and 8 and single insulated point. PS of algorithm F-2 contains one 13-shot cycle with path of the basin from 611 points, leaving on this cycle and single insulated point. PS of algorithm F-3 consists of two cycles with period 312 and single insulated point. The period's spectrum of algorithm F-4 (in parentheses repetition factor of cycle is specified) is  $T = 36(1)$ ,  $15(3)$ ,  $5(1)$ ,  $1(1)$  and has 538 points of the basin of the cycles.

The cycles of investigated algorithms F-1, F-2, F-3, F-4 have an important distinctive particularity: the behavior of the dynamic system before closing of the cycle (at the same time and on the basin paths if it exists) has a chaotic nature. At the same time the nonperiodic sequence generated by algorithm is a pseudorandom type. The set of points in PS combined into cycle can be named the pseudorandom cycle (PRC) if formed by algorithm nonperiodic process before closing the cycle has a chaotic feature in contrast to regular cycle which before closing has a feature like to the regular process. The pseudorandom

cycle corresponds to irregular motion in phase space and regular cycle — to regular one. Certainly in that and the other cases the dynamic system behavior on cycle is completely deterministic. The path of the pseudorandom cycle presents itself the deterministic set with chaotically following one for the other points of the states of the discrete dynamic system in all volume of algorithm's phase space. An analogue of the pseudorandom cycle of the discrete system is a strange attractor of continuous dynamic system.

In depending on values of system's parameters  $N_z \geq 3$ ,  $K_z$  and  $M$  in phase space algorithm F-1, F-2, F-3 it exists the variety of cycles of the various periods from which to each long ( $N \sim V_{PS}$ ) cycle before its closing corresponds the nonperiodic PRS with practically even distribution of generated numbers in given interval of the definitional domain  $p(x) \approx 1/M$  and with even distribution of conditional probability. Only for algorithm F-2 with the conditional probability function of distribution  $p(i+1, x_n/i, x_k)$  there are the forbidden transitions for even or odd numbers with depending on parity of the number on previous step. Take into considerations the processes only before closing the cycles, i.e. the nonperiodic segments formed by algorithm of PRS. These segments can have any length (as long as it pleased) according to choice parameter of algorithm and initial conditions. These segments can have any length (as long as it pleased) according to choice parameter of algorithm and initial conditions. So for algorithm F-1 at values of parameters  $N_z = 3$ ,  $M = 63$  the length of nonperiodic PRS is equal to  $N = 7.8317 \cdot 10^4$  ( $N/V_{PS} = 0.31$ ), for  $N_z = 5$ ,  $M = 63$  the length of nonperiodic PRS is equal to  $N = 3.3174 \cdot 10^8$  ( $N/V_{PS} = 0.33$ ), for  $N_z = 7$ ,  $M = 63$   $N = 1.676 \cdot 10^{12}$  ( $N/V_{PS} = 0.425$ ), for  $N_z = 9$ ,  $M = 63$  the length of nonperiodic PRS is more than  $5 \cdot 10^{12}$  step of the algorithm in the last case volume phase space is equal to  $V_{PS} = 1.56 \cdot 10^{16}$ . The long and superlong encoding sequences are need for ensuring the functioning complex navigational systems type NAVSTAR and GLONASS.

For comparison as example of PRS with uneven function of the probability distribution for generated numbers there are given the results of the study of the algorithm F-4. It is shown that for PRS formed by algorithm F-4 density of the probability distribution function  $p(x)$  falls off monotonous to beginning of the interval of the definitional domain  $[1, M]$ .

## 5. The Structured Complexity of Chaotic Binary Sequences

Practically all digital radio-engineering systems are using the binary signals. So it is sufficiently actual problem of the conservation of all particularities of the pseudorandom sequences binary numbers made by

way of clipping integer-value sequences formed with investigated algorithm of the Fibonacci type.

It is well known that the probability of the appearance of the block from  $k$  identical symbols in random binary sequence must follow the law  $p(k) = 1/2^k$ . In this case given binary sequence has good correlation features [10]. For random process of statistically independent events with equal probability this law must be follow probability of the appearance of any fragment consisted  $k$  binary symbols which are not certainly identical.

The purpose of the numerical experiment was to check an agreement of given regularities for binary PRS formed by variety algorithms with delay of Fibonacci type and qualification on this basis a quantitative criterion for estimation of structure complexity corresponding to binary PRS.

This problem was realized by checking the following positions:

- all possible fragments of the binary code by length  $k$  symbol are present in realization PRS;
- what is a probability of their appearance in realization of the sequences in consistent with law  $p(k) = 1/2^k$  that is true for ideal random process. It should be emphasized that exactly such checking, in particular, is provided by the standard of the crypto-operation Advanced Encryption Standard (AES) intended for statistical testing the code sequences which are applied for ensuring an confidentiality at transmission of information [14];
- an estimation of a structure complexity of binary PRS formed by algorithm Fibonacci-type with delay.

In the numerical experiment it was consecutively defined a frequency of the appearance in formed by algorithm realization from  $N$  members all possible fragments by length  $k$  taken from system of the full code with volume  $V(k) = 2^k$  where  $k = 2, 3, 12$ . A frequency of the appearance got in experiment for each  $i$ -that fragment of the full code  $n_i(k)/N$ ,  $i = 1, 2, \dots, 2^k$  were matched with probability  $p(k) = 1/2^k$  fragment by length  $k$  symbol in the sequences of the independent tests with equal probability. It have been calculated a dispersion

$$\sigma^2(k) = \frac{1}{2^k} \sum_{i=1}^{2^k} \left( \frac{n_i}{N} - \frac{1}{2^k} \right)^2 \quad (10)$$

and it was defined a standard deviation from this level  $1/2^k$  at given value  $k$ . In experiment the size of analyzed segment of the full code consecutively changed from value  $k = 2$  until  $k = 12$ . The lengths of analyzed realization of the sequences for all considered algorithms were defined by relation  $N = A \cdot 2^k$  where  $A$  was chosen from considerations of provision the

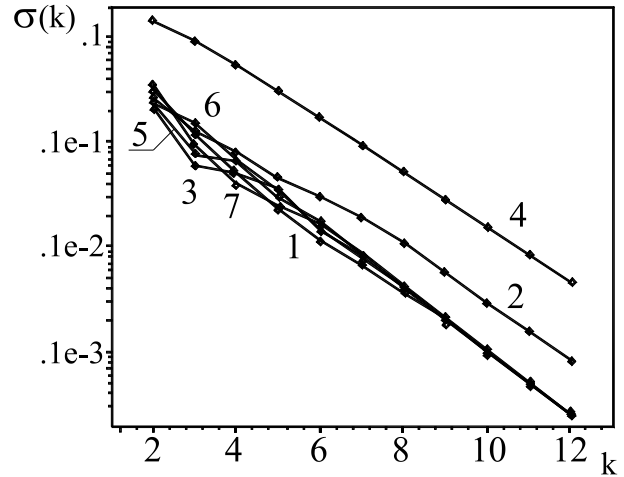


Fig. 2. The standard deviation probability of segments full code in realization binary PRS from law  $p(k) = 1/2^k$ .

necessary statistical representativeness of the samples. In the results presented below it were suggested that  $A = 100$  so as to in realization on each segment of the full code happened to not less 100 tests.

At the numerical analysis there was used following parameter's value of algorithms F-1, F-2, F-3, F-4:  $N_z = 16$ ,  $M = 255$ ,  $K_z = 9$ . For comparison also the standard generators of random numbers used in different program packages (Maple7, Mathcad and Pascal) were considered.

In Fig. 2 there are the graphs of the standard deviation of the appearance frequency all  $2^k$  variants segments of full code with length  $k$  from law  $p(k) = 1/2^k$  for algorithm F-1 (the curve 1), F-2 (2), F-3 (3), F-4 (4), generators RND Maple (5), Mathcad (6) and Pascal (7).

On the basis of obtained data it is possible to conclude that in sequences formed algorithm F-1 and F-3, the dispersion of the observed in experiment frequencies of the appearance segments of full code comparatively of level  $p(k) = 1/2^k$  practically coincides with corresponding features of the sequences formed by random numbers generators program packages Maple, Mathcad and Pascal. All this algorithms have the even distribution of probability generated numbers. Algorithm F-2 forming integer-valued sequence with even distribution of probability but with forbidden transitions for even or odd numbers on one step of the algorithm has the dispersion of probability of the appearance fragments full code in 2-3 times more. Nonuniform distribution generated numbers in the sequences as in the case for algorithm F-4 brings to essential (on order) deflection from uniformities of the appearance fragments full code in realization formed binary process.

On the basis of the numerical experiment results it was defined the average value of standard deviations

for all analyzed full code systems ( $k = 2, 3, \dots, 12$ ):

$$\sigma_{av} = \frac{1}{11} \cdot \sum_{i=2}^{12} \sigma_i(k). \quad (11)$$

Let us define  $K_{SC} = 1/(1 + \sigma_{av})$  as factor characterizing of structure complexity PRS with respect to the complexity of purely random binary sequence.

The obtained quantitative values of the factor  $K_{SC}$  for all analyzed algorithms were provided in Table 1. These data show that the structure complexity of the pseudorandom binary sequences formed by integer-valued algorithms with delay on the basis of the Fibonacci-type mapping with the following clipping which have practically even probability distribution of generated numbers  $p(x)$  and close to even distribution of conditional probability (probabilities of transitions) does not differ greatly from structure complexity of purely random sequences. Exactly in the same way either as from complexity of the sequences generated by certificated generator of random numbers.

The algorithms with high structure complexity must have the correlation properties closed to corresponding properties of the random process. For all analyzed algorithms of the Fibonacci type with operation of returning generated numbers in interval of the definitional domain in Table 1 are brought estimations of level  $R_{max}$  of collateral surges of aperiodic autocorrelation and cross correlation functions. In the numerical experiment the aperiodic correlation functions were defined on 100 non-overlapping segments with length  $N_{cod} = 128$  (that corresponds to the standard IS-95 for communication CDMA systems) consecutively generated by algorithm without any selection including without selection on code's balance. The obtained levels of correlation function's collateral surges of segments of binary PRS generated by algorithm F-1,F-2,F-3 are rather good correspond to the correlation function's of collateral surges of the random sequences [10]. So the structure complexity of the sequences generated by designed chaotic algorithms practically coincides with the complexity of the really random sequences. Such sequences can be used as extending signals in radio-engineering and navigation systems with noise-like signals.

## 6. The Fractal Analysis Methods of Chaotic Algorithms

For efficient application of algorithm for fractal processing it necessary to transform the algebraic object-sequence of numbers or signs in the form of

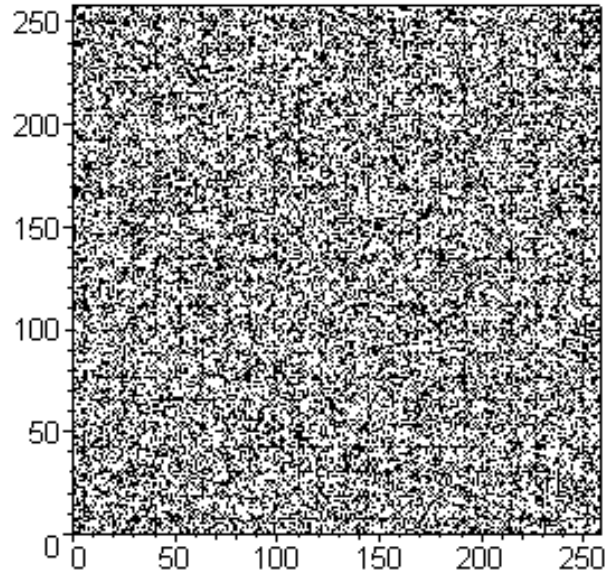


Fig. 3. The projection of phase space for chaotic algorithm with delay.

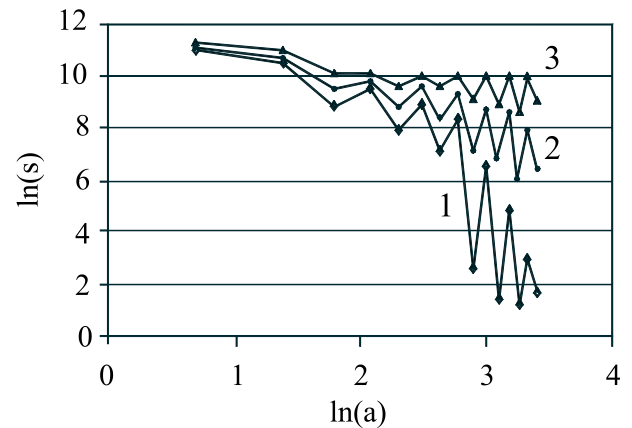


Fig. 4. Fractal signatures of twin maps (the square – map of  $(x_n; x_{n+1})$ , the circle –  $(x_n; x_{n+2})$ , the triangle –  $(x_n; x_{n+9})$ ).

graphic image. As geometric images characterizing the characteristics of chaotic algorithm it is possible to choose the incremental image on planes of the recurrence sequence members as well as projection of multidimensional phase space (PS) of the chaotic algorithm on to one of the coordinate planes. The example of such PS projections for algorithm with parameter  $N = 30000$  is presented in Fig. 3.

The measurement of fractal characteristics was produced on the brightness field of images. In case of two-dimensional field can be used two methods. The first method of "running window" allows to get the dependency  $S = f(\delta)$  where  $S$  – measured parameter,  $\delta$  – a size smoothing window. The second method of the measurement of local dispersion dimensionality concluding in measurement of the

Table 1.

Algorithm	$K_{SC}$	$R_{\max}\sqrt{N_{cod}}$ , ACF	$R_{\max}\sqrt{N_{cod}}$ , CCF
F-1	0.95	1.3-3.6	1.5-4.0
F-2	0.94	1.6-3.6	1.6-4.1
F-3	0.96	1.14-4.2	1.5-4.6
F-4	0.73	2.9-6.9	1.75-5.7
RND Maple	0.96	-	-
RND Mathcad	0.95	-	-
RND Pascal	0.95	-	-
M-sequence [10]	-	0.7-1.25	1.4-5.0
Segments of M-sequence [10]	-	1.45-4.1	-
Random sequences [10]	1.0	2.1-3.5	2.1-3.5

brightness dispersions of the small area of image on two scales. This method allows to get the spectrum of fractal dimensionality on image [15].

The results of the computer calculation of fractal signatures  $\ln S = f(\ln a)$  were presented in Fig. 4., where  $S$  – brightness parameter of the graphic image (Fig. 3),  $a$  – side of window (the measuring window was square and relative size of window was changed from 3 before 30 pixels).

From Fig. 4 one can see that all the signatures have parts with different dominant inclinations. These facts are characterized power of statistical interconnection between corresponding pairs members of recurrent chaotic sequences. The analysis shows that fractal signatures of algorithm’s mapping with good level of mixing (that means a weak statistical interconnection between pairs members of recurrent chaotic sequences) are characterized by a smaller diversity and practically identical inclination.

For characteristics estimation of the fractal chaotic set of points on PRC let us to limit by the analysis of Euclidean  $D$  and correlation dimensions  $D_2$ . The numerical experiment was organized for small values of algorithm’s parameter  $N_z = 4$ ,  $M = 11$  with the length at investigation PRS from  $N = 500$  numbers that has principle importance for estimation the majority characteristics of pseudorandom cycles. On increasing a dimension of algorithms the behavior of the discrete DS becomes greatly complicated and improves the statistical features of formed PRS.

The estimation of correlation dimension  $D_2$  at investigation pseudorandom cycle possible to give on the basis of the calculation correlation integral  $C(l)$  given on set of the distances  $l$  between all pairs of state vectors on cycle in FS and drawing the dependencies  $\lg(C(l)) = f(\lg(l))$  shown in Fig. 5 and determinations on its of angular factor of the rectilinear area [16].

For algorithm F-1 with parameter  $N_z = 4$ ,  $K_z = 2$ ,  $M = 11$  the correlation dimension for the set of points on cycle with initial vector  $R_0(8, 6, 7, 1)$  (the curve

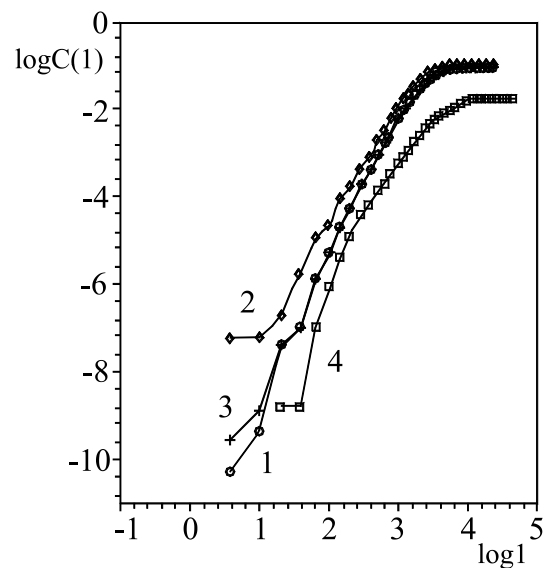


Fig. 5. The dependency  $\log C(l)$  from  $\log l$  for algorithms F-1, F-2, F-3 and F-4.

1) is equal to  $D_2 = 3.3$ . The obtained values are in good agreement with geometric dimension  $D = 4$ ,  $D_2/D = 0.83$ . The values of last relations can serve as the feature of a degree to homogeneity of the filling cycle by points of the full volume PS. It was shown by the analysis that the nonperiodic PRS with the length  $N = 14030$  with function of distribution closed to even corresponds to the investigated cycle with initial vector  $R_0(8, 6, 7, 1)$ .

The curve 2 is got for ensemble point to paths of the pool and cycle in phase space of the algorithm F-2 ( $N_z = 4$ ,  $M = 11$ ,  $R_0(1, 1, 1, 1)$ ,  $N = 500$ ). The linear area of this graphics has smaller sloppings that corresponds the correlative dimensionality  $D_2 = 3.0$ . The curve 3 on Fig. 5 corresponds to the logarithm of correlative integral for pseudorandom cycle with  $R_0(1, 6, 6, 7)$  of the investigated algorithm F-3,  $N_z = 4$ ,  $M = 11$ ,  $N = 500$ . The curves

1 and 3 for function  $\log(C(l)) = f(\log(l))$  in Fig. 5 almost in accuracy repeat each other and have lengthy rectilinear area with slopping  $D_2=3.3$ . This allows to get the quantitative estimation to homogeneity of the filling space by points of DS states on pseudorandom cycles. It necessary notes that algorithms F-1 and F-3 correspond to PRS with good statistical and correlative characteristic especially for delay parameter  $N_z$  more 5.

For cycle of the algorithm F-4 (with parameters  $N_z = 4, M = 17$  length to sequences  $N = 500$ , initial radius-vector  $R_0(7, 14, 6, 15)$ , period  $T = 613$ ) the dependency  $\log(C(l)) = f(\log(l))$  (the curve 4 on Fig. 5) has not clearly expressed rectilinear area. This means that correlative integral has divergence from law  $C(l) \sim l^{-D}$  and consequently the points of this pseudorandom cycle are located irregularly in PS.

For estimation of complexity degree of the chaotic process formed by algorithm it is necessary to define homogeneity of attractor in PS on all scale of discrete time. The determination of correlative dimensionality of attractors requires the big computing resource (especially for DS with high dimensionality). So it makes sense to research the structured characteristics of the projection of DS motion paths in PS on one of the directions in this space.

The fractal analysis can be applied not only to the points of chaotic set in multidimensional PS but also to one-dimension set numbers of the realization of PRS. The determination of correlation dimension on the standard methods applying to one-dimension ( $D = 1$ ) chaotic array from  $N = 1000$  numbers PRS formed by algorithm F-1, F-2, F-3, F-4 at parameter  $N_z = 16, M = 21$  has given the following results. For all tested algorithms the values of correlation dimension were found order  $D_2 = D_2/D = 0.91 \div 0.96$  including the generator of random numbers RND (Maple) (for  $M = 21$ ). These data of  $D_2/D$  is indicated of good homogeneity of the filling up the interval  $[1, M]$  by generated numbers. These results are confirmed by analysis of the one-dimension probability of distribution for numbers in the sequences. But on the basis these data it is impossible say anything about structured complexity of PRS. For this purpose there was fulfilled the study of the local structure PRS on the basis of the analysis fractal geometry [9].

The random sequence integer numbers it is possible to consider as discrete topology of the complex geometric relief ("coast line"). For estimation of the geometric structured complexity there were made research by consecutively changing the distance between nearby points of such relief in window with the specified scale. In other words on the basis of realization data PRS with the length  $N$  let us go to

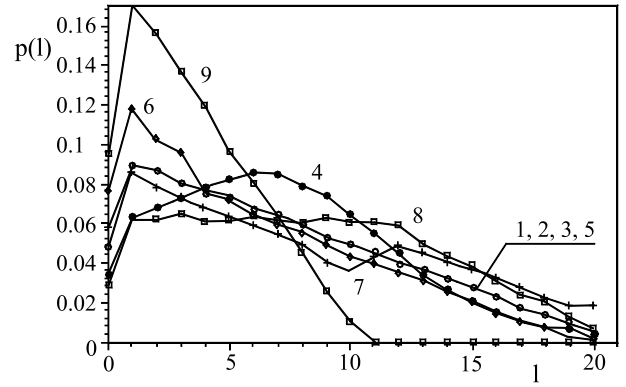


Fig. 6. The probability of the differences of numbers  $l = |x_n - x_{n+1}|$  in realization of the sequences.

analysis of the algebraic sequence

$$\{y_n = |x_n - x_{n+1}|\}, \quad n = 1, 2, \dots, (N - 1),$$

$$y_n \in [0, (M - 1)]. \quad (12)$$

In accordance with the method of correlation integral's calculation let us calculate the number  $N(l)$  of the realization of identical events  $y_n = l$ , where  $l = 0, 1, 2, \dots, (M - 1)$  in the sequences from  $(N - 1)$  of the members and draw the curves of the frequency of realization such events  $p(l) = N(l)/(N - 1)$  as function of  $l$  (Fig. 6).

The calculations are executed for PRS with length  $N = 50000$ . The curve 1 corresponds to the algorithm F-1 with parameter  $N_z = 16, K_z = 9, M = 21$ . This curve repeats nearly in accuracy the corresponding theoretical dependence for the sequence of statistically independent numbers with an equal probability of realization -  $p_{st}(l)$  which was taken as the standard one. As the last one it was possible to use experimental values  $p(l)$  for PRS in the case of its vicinity to theoretical, for instance, for PRS formed by generator of random numbers RND or algorithm F-1. The total absolute value of the deflections from standard for values  $p_i = p(l_i)$  which have been got for analyzed sequence would specified as  $s = \sum |p_i - p_{st.i}|$ . It was possible to take the value  $S = 1/(s + 1)$  as the structure's complexity for this process  $\{x_n\}$ . The curves 2, 3 and 5 have been got at analysis algorithm F-2, F-3 with same large dimension PS ( $N_z = 16, M = 21$ ) and generator of random numbers RND (Maple) with  $M = 21$ , also were a closely approximated from standard curves. The curve 4 corresponds to the algorithm F-4 with uneven probability distribution of generated numbers  $p(x)$ .

The graphs 6,7,8 and 9 are constructed for modified PRS algorithm F-1 for the reason modeling of the discrete processes with variety types functions of distribution  $p(x)$  (the average of distribution -  $x_{aver}$ , standard deviation -  $\sigma$ , coefficients of skewness -  $\gamma_1$  and kurtosis -  $\gamma_2$ ) and interval of autocorrelation  $\tau_{cor}$ . The curves 4, 6,7,8 and 9 differ noticeably

Table 2.

Plot Fig. 2	Algorithm	$p(x)$ of tested PRS				$\tau_{cor}$	Distinction $p(l) = f(l)$ out of standard	
		$x_{aver}$	$\sigma$	$\gamma_1$	$\gamma_2$		$s$	$S$
1	F-1	10.9	6.07	$1.08 \cdot 10^{-2}$	-1.21	1	$4.50 \cdot 10^{-2}$	0.96
2	F-2	11.0	6.05	$-1.97 \cdot 10^{-3}$	-1.20	1	$4.70 \div 10^{-2}$	0.96
3	F-3	11.0	6.07	$1.85 \cdot 10^{-2}$	-1.25	1	$4.80 \cdot 10^{-2}$	0.96
4	F-4	8.29	5.04	-1.21	-0.40	1	0.22	0.82
5	RND	11.0	6.06	$1.0 \cdot 10^{-3}$	-1.20	1	$4.40 \cdot 10^{-2}$	0.96
6	F-1 modify	11.0	5.55	$8.79 \cdot 10^{-2}$	-1.21	2	$1.85 \cdot 10^{-1}$	0.85
7	F-1 modify	11.0	6.81	$-1.02 \cdot 10^{-2}$	-1.43	1	$2.12 \cdot 10^{-1}$	0.83
8	F-1 modify	7.32	5.85	0.86	-0.43	-	$-1.46 \cdot 10^{-2}$	0.87
9	F-1 modify	11.1	5.26	$-2.14 \cdot 10^{-2}$	-0.88	10	0.63	0.62

from standard. This fact indicates of high informative significance of the proposed method of evaluation the algorithm's structure complexity by means of constructing the graphic of the relative frequency appearance of the difference value between any nearby numbers in realization PRS  $p(l) = f(l)$ . The given calculated method does not require the great computing resource in contrast with methods of statistical, correlation and fractal analysis.

In Table 2 there are given the results of numerical experiments for all tested algorithms (the parameters of Fibonacci type's algorithm:  $N_z = 16$ ,  $M = 21$ , the realization's length  $N = 50000$ ).

From given data it is seen that all three algorithms F-1, F-2 and F-3 on the basis of the Fibonacci mapping as well as certified generator of random numbers RND are demonstrating high level of structured complexity for the formed sequences. At changing the functions of the probability distribution for generated numbers  $p(x)$  and factor of correlations the suggested methods of estimation the structure complexity allows to effectively fix the corresponding changing of statistical characteristics PRS.

Thereby it was shown that the fractal signatures calculation of recurrence chaotic sequences and its graphic images allows quantitatively to estimate the efficiency of the mixing mechanism and the degree of statistical links between pairs of the chaotic sequence members that finally defines the complexity of chaotic generating algorithm.

## 7. The Chaotic Noise-like Signals in Digital Radio Systems with Spread Spectrum

The important requirement presented to modern

telecommunication systems is a provision of noise-immunity, electromagnetic compatibility and confidentiality [17]. The electromagnetic compatibility of the operation wireless communications systems increases to account of the spectrum expansion and generation noise like continuous radiations with low spectral density. The protection of sent information from unauthorized access is provided by coding of messages by means of long nonperiodic pseudorandom sequences and to account of the dynamic change at time of the encoding sequences [18].

At development and making of the systems with code division channel (CDMA) it is important a choice of mathematical encoding algorithm generating big ensemble PRS. Formed PRS must have necessary statistical and spectral characteristic as well as good auto- and cross relative features. The special requirements are presented in respect of large volume of the ensemble orthogonal PSP that required for simultaneous and stable functioning of the many users in the general spatial zone. The mathematical algorithms must generate the ensemble of the statistical independent pseudorandom codes of large duration and high structured complexity for providing of confidentiality at transmission of information [19].

In modern digital radio engineering systems the spread spectrum technology is often used for forming the wideband signals. For increasing of the information capacity as spreading function the various complex signals closed to noise-like process are chosen.

As a rule the broadband signals are formed due to expansion of the frequency band of the information signal and (or) due to expansion the spectrum of carrying. The expansion of the signal frequency band is usually given by such modulation of the carrier which forms the modulated signal with frequency band more than of the modulating function. The typical example of the signal with expansion of the frequency band is the frequency modulation with large

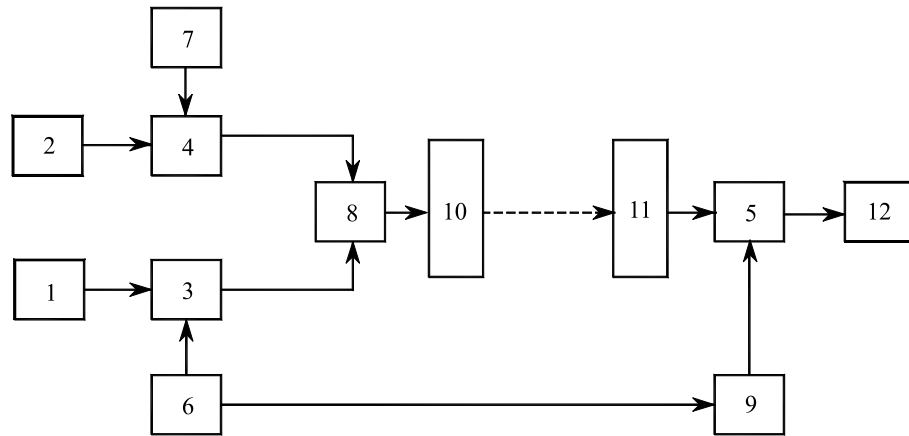


Fig. 7. The block scheme of the experiment on noise-immunity cross channel of models of the terminal, UHF-generators of the signal and the interference – 1 and 2 accordingly; the phase modulators FM<sub>1</sub>, FM<sub>2</sub> and FM<sub>3</sub> – 3, 4 and 5; the coders – 6 and 7; UHF-adder – 8; the controlled line of the delay – 9; the transmitting and receiving UHF-antennas – 10,11; the spectrum analyzer – 12.

modulation index.

The extension of the frequency band resides also to the digital telecommunication channels with additional noise immunity coding because introducing excess symbols at keeping unchangeable rate of information transmission requires to reduce duration of each symbol. At the same time the frequency band of transmitted coded signal enlarges.

As the numerical analysis shown the discrete chaotic algorithms have a good statistical and correlative characteristics and a structural complexity, fractal characteristics enough close to properties of the random process [20]. This allows to use its in various radio engineering systems with spread spectrum and code division. At the same time the system signal volume formed on the basis of such sequences greatly exceed the volumes of traditionally applicable pseudorandom sequences [21].

The efficient way of the spectrum expansion of the signal is its phase shift-keying. This is realized on the basis of the discrete phase manipulator. For ensuring the broad frequency band most efficient is application of  $\pi$ -modulator in which are realized two states corresponding to absence the phase shift (zero shift) and phase shift on  $\pi$ . Such modulators are effectively realized on scheme with switching  $p - i - n$  diodes.

For efficiency estimation of designed encoding algorithms the model of the modem for broadband communications system with noise like signal on the basis of spread spectrum was designed. The basic element of the scheme is UHF bridge phase shift module with fixed phase shift  $\varphi = \pi$  and generator of digital chaotic sequence realizing operation of such algorithm on the basis of the high-speed digital technology. The cross channel noise-immunity of radioterminal's model on the basis noise-like carrier in which for transmiton of information were used binary

chaotic signals was experimentally researched (Fig. 7). The coders realized one of the generation algorithms of the class chaotic signal type described above.

For study the division efficiency of nearby distance channels the variable temporary delay between the phase shift keying noise like signal at coder's output and the signal at the decoder's input was hardware entered. The code formed by digital generator of the chaotic encoded sequences was given at demodulator's input with delay relative to moment of the presenting it at input the modulator. The necessary discrete delay was realized hardware at passing of the signal through consequent system of the digital integrated circuit provided step-like change of delay this signal relative signal at input of the demodulator.

Spreading of the spectrum of the signal from UHF-generator (1) was produced by means of microwave phase manipulator FM<sub>1</sub> (3). UHF signal on average frequency  $F_{av}$  entered at input FM<sub>1</sub>. The control was realized by chaotic binary sequence of pulses from coder (6). As a result at output FM<sub>1</sub> (3) there is the noise signal with continuous spectrum.

Microwave signal with frequency  $F_m$  proceed at the input of the phase modulator operated by chaotic binary sequence pulse from digital coder. At the output the signal had continuous noise like spectrum. To this signal it was added a noise signal and the total one was radiated and was taken by the receiving antenna. The convolution of the accepted noise like signal with chaotic binary sequence was produced in the receiving module. The decoding chaotic binary sequence was formed by digital generator and was identical to chaotic binary sequence in modulator.

The two types of the interferences were used in experiment on determination of noise-proof: the sinusoidal interference close on frequency to transmitted UHF-signal and the broadband interference coordinated on spectrum with

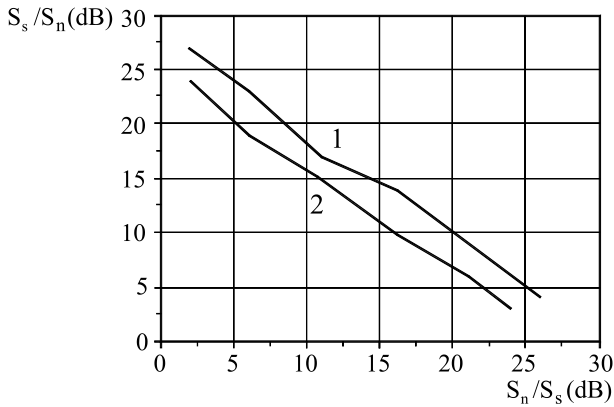


Fig. 8. The signal/interference ratio ( $S_s/S_i$ ) at the output of receiving device depending on level correlations of interference and information signal at the input of receiver ( $S_i/S_s$ ) for two types of the interference: narrow-band (1) and broadband (2).

transmitted signal. The broadband interference was formed by means of microwave phase manipulator FM<sub>2</sub> (4). The control by FM<sub>2</sub> (4) was realized by separate coder (7) with the same clock rate as coder (6) of the transmitter. The encoding sequences both coders (6) and (7) are different and uncorrelated at time. The experiment was executed at synchronism of the encoding sequences for modems of the transmitter and receiver that was provided by using of controlled delay (9). The backward coherent compression on frequency of the received signal was produced by modem (5) in the receiver.

As noise-proof criterion there was taken the excess of the convolute signal at the output receiving device (12) above noise background depending on ratio signal/interference at the input receiver.

At influence of the narrow-band interference the total spectrum of signal and interference at the input receiver has the form of the continuous broadband noise corresponding to the received spread spectrum signal and peak of sinusoidal interference. In receiver decoder FM<sub>3</sub> provides convolution and separation of the useful signal. Simultaneously there is a power's distribution of the narrow-band interference extensively on spectrum at the same time transforming the narrow-band interference in to noise pedestal on which the convolute information signal raises above.

The results of the measurement of ratio signal/interference ( $S_s/S_i$ ) at the output receiving device depending on interference of correlations of information signal and interference at the input receiver ( $S_i/S_s$ ) for two types of the interference: narrow-band (1) and broadband (2) are presented on Fig. 8. The maximal noise-immunity for communications systems with spread spectrum is defined by ratio signal/interference at the input receiver as the reconstruction of transmitted

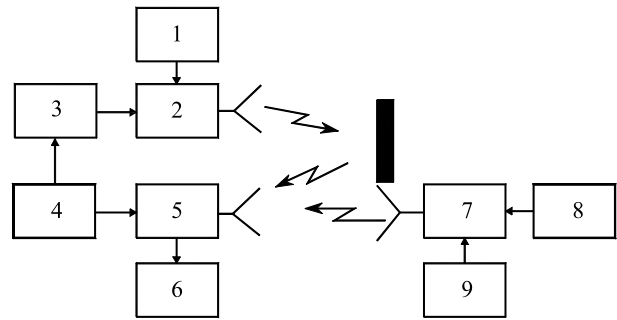


Fig. 9. The block scheme of channel model with noise like carrier. The generator of sinusoidal UHF signal – 1,8; the phase modulator – 2,7; the source to encoding chaotic sequence (the expansive function) – digital generator – 3,9; the block of the discrete delay – 4, the phase demodulator – 5; the spectrum analyzer – 6.

information becomes the impossible at given average time.

The experimental results show that for both types of the interference the noise-immunity was equal to 25 dB. At transmission of information the continuous noncyclic encoding chaotic sequence realized for practically unlimited time interval is used. By such way the analogue of the mode of the dynamic change the codes for the total time data communication is realized. According to Shannon in this case it is practically excluded the possibility of the cryptographic decoding of the message [22]. This result is indicative of potential resources of using the system with spread spectrum on the basis of the chaotic codes in multistationing mobile communications systems with high degree of confidentiality.

Similarly the receiving-transmitting module (Fig. 9) realizing the single distance channel of radar on the basis noise-like carrier can be designed. The module contains: generator of sinusoidal signal UHF-range (1), the digital generator forming encoding chaotic sequence (3), the phase modulator (2) and demodulator (5), the spectrum analyzer connected in different points of the scheme (6), block of the delay (4).

For study the division efficiency of neighboring distance channels the variable temporary delay between the phase shift keying noise like signal at coder's output and the signal at the decoder's input was hardware entered. The code formed by digital generator of the chaotic encoded sequences was given at demodulator's input with delay relative to moment of the presenting it at input the modulator. The necessary discrete delay was realized by hardware at passing of the signal through consequent system of the digital integrated circuit provided step-like change of delay this signal relative signal at input of the

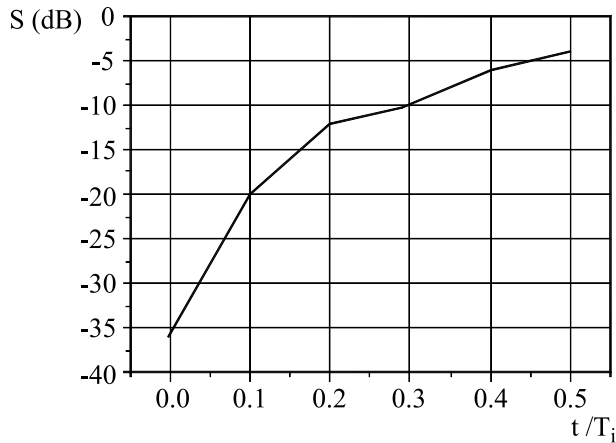


Fig. 10. The signal level at demodulator's output depending on time of the shift between encoding and decoding chaotic sequences in neighboring distance channels.

demodulator.

The obtained results show that shift between encoding and decoding chaotic sequences for time  $\tau$  equal to half of duration modulation pulses  $T_i$ , i.e. at transition in neighboring distance channel results to practical disappearance carrying in restored signal [23]. This is evidence of good uncoupling ( $\approx 30$  dB) between nearby distance channels comparatively each other (Fig. 10).

## 8. Conclusion

On the basis of nonlinear systems with chaotic dynamics it was designed and investigated statistical, correlative and fractal characteristics of the pseudorandom discrete signals with high structure complexity. The complex signals of such type have the high information capacity and can be effectively used in different radio-engineering systems including telecommunication as well as in radiolocation and navigation.

It was shown that on the basis algorithms with nonlinear dynamics can be designed the large ensemble of the chaotic binary codes that allows practically to realize code division of the large number abonents. The digital broadband communication system with spread spectrum on the basis of the large chaotic binary code ensemble has the high degree of noise proof factor and the high degree of confidentiality at communication under conditions of the complex electromagnetic situation, influences of the strong interferences and multipaths of the signal spreading.

The experiments were executed in the microwave band on simulating digital radio channel with the spectrum's expansion by means of using the phase modulator-demodulator. As expanding functions for

phase modulation and demodulation of carrying the pseudorandom digital signals on the basis of the binary sequences generated by proposed chaotic algorithms were used.

It was shown that in such telecommunication channel there was realized an efficient expansion of the signal spectrum transformed it to broad band noise like carrier. So the information spectral components absolutely do not stand out of the general noise background.

At simulating of division efficiency of neighboring channels it was shown that correlation properties of complex chaotic signals provided the low value factor of correlations between neighboring distance channels that was indicated of good uncoupling of neighboring distance channels.

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