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Dynamics of Electric Fields and Gravitation

Contents

Abstract

The behaviour of an electric field in a gravitational field is analyzed. The electric field of a charge accelerated in free space is studied using the equations of STR, and it is found to be curved due to the acceleration of the charge. The electric field of a charge located statically in a gravitational field curves too due to the fall of the electric field in the gravitational field. In both these cases, the stress force created in the curved electric field acts as a reaction force which leads to the creation of electromagnetic radiation – both by a charge accelerated linearily in free space, and by the electric field falling in a gravitational field. Maxwell equations are used to show how the energy flow of the falling electric field serves as the source of the energy carried by the radiation.

1. Introduction

The electric field (EF) of an electric charge is a very fundamental phenomenon. Since the robust rules for the behaviour of this EF where stated by Maxwell fifteen decades ago, the phenomenon is studied in detail, and it got a very broad use in almost each field in our life – actually, one cannot imagine life in the present epoch without the intensive use of electrical devices in any field of our culture.

However, very little is said about the nature of the EF – is it just a convenient mean to describe phenomena connected with electric charges, or it is an independent physical entity that should be dealt with as a physical entity. Einstein [1] argued very clearly that an EF is an independent physical entity, that should be treated as such. Landau and Lifshitz [2] also support the approach that an EF is an independent physical entity. In the above mentioned book [1, pp 256, 260] Einstein writes: "We have two realities: Matter and field. There is no doubt that we cannot at present imagine the whole of physics built upon the concept of matter as the physicist of the early nineteenths century did. For the moment we accept both the concepts. . . . The theory of relativity stresses the importance of the field concept in physics. But we have not yet succeeded in formulating a pure field physics. For the moment we must still assume the existence of both: field and matter". Einstein expected that a unified theory will be able to describe matter points as concentrated fields, but he did not succeed in reaching this goal. In a recent paper, Wilczek [3] argues that this goal is supposed to be achieved in electroweak interactions theory. However, when one considers classical (relativistic, non-quantum) physics, one should consider fields and charge points as two independent entities, and treat them on an eqaul level. In this regard, the approach of physicists that cannot accept the idea that a field is an independent entity, limits the ability to understand correctly physical phenomena. Physicists intuitively accept the idea that radiation fields (the part that falls with the distance like 1/distance) are independent entities that lost their dependence on the source, once they were induced on space. They should also accept the idea that an EF (that falls with the distance like 1/distance^2) are also independnet physical entities.

In the present work we are going to analyze electrocmagnetic phenomena, for which the only satisfactory explanation is the one based on the approach that electric fields are independent physical entities, and their behaviour as independent entities creates the important phenomenon named "electromagnetic radiation".

For example, when an electric charge is accelerated in free space, the equations that describe its EF are calculated by STR formalism. It is found that its EF becomes curved [4], and for the simple case of a linear aceleration (described in the literature as a hypebolic motion), the equation for the characteristic radius of curvature, R_c, is: $R_c = c^2/a$, where a is the linear constant acceleration. In a curved field a stress force is created, and the equation for this force density, f_s, is:

$$f_s = \frac{\mathbf{E}^2}{4\pi R_c}. \qquad (1)$$

This stress force acts as a reaction force on the accelerated charge, and it opposes the acceleration. The external force, which accelrates the charge has to perform an extra work in order to overcome this reaction force, and this extra work is the source of the energy carried by the radiation [4, 5, 6].

When a charge is located in a gravitational field(GF), the situation is different. The charge, which is supported against free fall, is static, and certainly cannot create any dynamical effects that can lead to the creation of radiation. However, the EF of the charge is not supported with the charge since it is emanated on the space around the charge, and is free in its response – the mass (energy) density of the EF is subject to gravity, and it falls in the GF. Due to this fall, the EF curves, and a stress force is created in this curved field. This stress force acts between the static charge and its curved falling electric field, "trying" to prevent the EF from its free fall. The acting agent in this case is the GF that causes the fall of the EF. In order to overcome the delay of the free fall of the EF, the GF has to perform an extra work on the falling EF – this work is the source of the energy carried by the radiation created by the falling EF.

Thus, although the charge is static, and cannot create any dynamical effect like the emission of radiation, the EF of the charge acts dynamically, and electromagnetic radiation is still created in this case. The source of the energy is the gravitational energy of the system, which becomes lower.

In Section 2 we analyze the case of a charge accelerated in a free space. In Section 3 we describe the EF of a charge located in a homogenous GF. In Section 4 we describe the EF of a charge located in a central GF. In Section 5 we use Maxwell equations to discuss the dynamical effects in these cases, which give rise to the creation of radiation, and we summarize in Section 6.

2. A linear acceleration

The case of a charge accelerated linearly in free space is described in details by Fulton and Rohrlich [7], and we shall use here their results. The equations they find for the electric field of such a charge (given in cylindrical coordinates, ρ, z, ϕ) are:

$$\mathbf{E}_\rho = \frac{8e\alpha^2\rho z}{\xi^3} , \qquad (2)$$

$$\mathbf{E}_z = -\frac{4e\alpha^2}{\xi^3}[\alpha^2 + c^2t^2 + \rho^2 - z^2] , \qquad (3)$$

where $\xi^2 = (\alpha^2+c^2t^2-\rho^2-z^2)^2+(2\alpha\rho)^2$, and $\alpha = c^2/a$ is the charge location at $t = 0$ (the turning point of the charge trajectory when plotted in Rindler coordinates [8]). α is also the characteristic radius of curvature of the lines of the EF of the accelerated charge. The same expressions for the electromagnetic fields were also found by Gupta and Padmanabhan [9], who calculated first the electromagnetic fields in the rest system of the charge, and then transformed them to a system moving in a hyperbolic motion relative to the charge. They also show that by using appropriate transformations to the system of reference of the retarded variables, they recover the equations given in the textbooks [2, 10, 11] for retarded potentials.

Using Eqs. 2, 3 , Singal [12] calculated the equations for the electric field lines for this case, and draw them in a figure. A similar figure is displayed here:

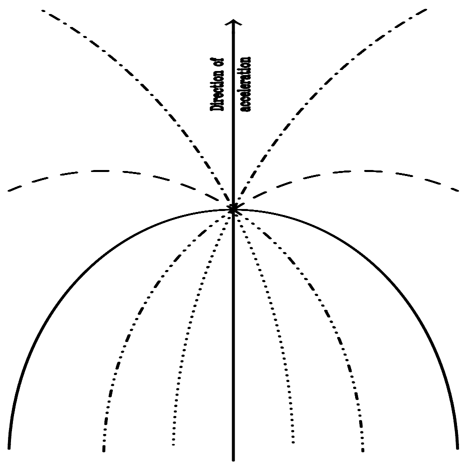

Fig. 1. The electric field lines of a charge accelerated linearly in free space.

The equations for these field lines where calculated by Fulton and Rohrlich using the retarded potentials method, namely, the electric field at each field point x at time t, is determined by the charge when it was located at the point x', at an earlier time t', where

$t' < t$. The relation between these coordinates is: $x - x' = c(t - t')$. The motion of the charge after time t', did not influence the field at location x, at time t. Namely, once the electric field was induced on the space around the charge, it is not influenced by the charge any more – the field is independent from the charge that induced it, and it is an independent physical entity that should be treated as such.

It is interesting to compare this figure with the figure that displays the EF lines of a charge moving with a constant velocity in free space. Here also the field is calculated by using the retarded potential method.

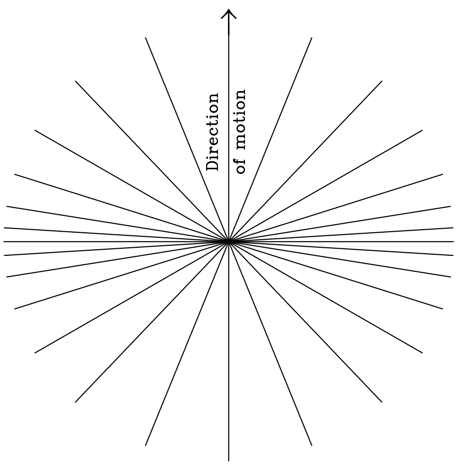

Fig. 2. The electric field lines of a charge moving uniformly in free space.

We clearly see that in this case the field lines are straight lines, although their space distribution is not spherically symmetric around the charge. Here also, this distribution of the field lines is created because the electric field is not influenced anymore by the charge, once it is induced by the charge on space. Hence, the distribution of the field lines is not spherically symmetric. Since the velocity of the charge is constant, the ratio between the charge velocity and the velocity of the field expansion is constant, and the field lines are straight lines. In the case of the linear acceleration discussed above, this ratio between the charge velocity and the velocity of the field expansion is not constant, and hence the field lines curve.

We summarize this section by noting that the independence of the EF from the behaviour of the charge that induced it, is the main feature that enables the creation of the electrocmagnetic radiation by the accelerated charge, through the stress force created in the curved field, that acts as a reaction force.

3. A charge in a homogenous gravitational field

After we defined the conditions needed for the creation of radiation by an accelerated charge, we turn to the case in which a charge is supported statically in a homogenous GF.

The equations for the EF of a charge supported in a homogenous GF of strength g, were calculated by Rohrlich [13] using cylindrical cordinates $(\rho,\ z,\ \phi)$:

$$\mathbf{E}_\rho = \frac{8e\alpha^3\rho u^2}{\xi^3}, \tag{4}$$

$$\mathbf{E}_z = -\frac{4e\alpha^3 uu'}{\xi^3}[\alpha^2(1-u^2)+\rho^2], \tag{5}$$

$$\xi^2 = [\alpha^2(1-u^2)-\rho^2]^2 + 4\alpha^2\rho^2, \tag{6}$$

where $u^{-1} = \cosh(\sqrt{(1-gz)^2-1})$, and $u' = du/dz$. $\alpha = c^2/g$ is the radius of curvature of the EF close to the location of the charge. From these equations one can calculate the derivative of z with respect to ρ for the field lines of force (see Singal [12]). This expression cannot be integrated analytically, but still, using this derivative, the lines of force can be calculated numerically, yielding Fig. 3.

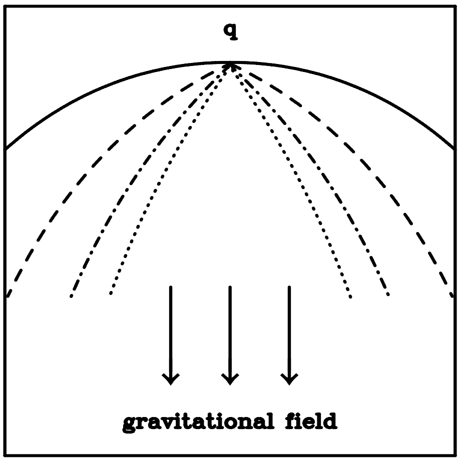

Fig. 3. Curved field lines of a charge supported in a homogenous gravitational field.

Usually, when considering the question whether an electromagnetic radiation is created in this case, the immediate response of people is: "The situation is static, no work is performed in a static situation, and hence no radiation can be created". However, the situation is not static, but rather a steady state one, in which the EF of the static charge falls in the GF. If one watches a certain point in the space around the charge, he will always see there the same field strength. However, this field is constantly "streaming" through this point, like a constant point in a streaming river – the picture at each point is always the same, but the constant flow is a dynamical effect. We know that an EF of a charge is an independent physical entity, and once induced on space, its behaviour does not depend any more on the behaviour of the charge that induced it. The charge is supported and remains static. The EF of the charge is induced on space around the charge. The mass (energy) density of the EF is subjected to gravity, and it falls in the GF. Due to this fall, the EF becomes curved (as can be observed in Fig. 3), and a stress force is created in the EF. This stress force interacts with the static charge, and this is actually a reaction force that acts back on the falling EF, braking it from being a free fall. In order to keep the EF in a free fall, the GF has to perform an extra work on the EF. This extra work is the source of the energy carried by the radiation. Since this extra work is performed by the gravitational source that creates the GF, this source loses energy - namely, the gravitational energy of the system becomes lower. The energy carried by the radiation is created at the expense of the gravitational energy of the system.

It comes out that indeed, the supported charge does not radiate (because it is static and no work can be performed on a static object). However, the configuration in which the charge is supported in a GF does radiate – the work that creates the radiation is performed by the source of the GF, and it is performed on the EF, against the reaction (stress) force, to maintain its free fall. The energy loss, carried away by the radiation, is supplied by the object that performs the work – the gravitational field. In this picture, the moving entity is the EF, and the work is performed on this moving entity.

4. A charge in a central gravitational field

The equations for the EF of a charge located in a Schwarzschild GF were calculated by Hanni and Ruffini [14], and the potential fields of point charges in general Schwarzschild fields were calculated by Linet [15], who improved the method of Copson [16]. Copson calculated the electric potential of a point charge located in a Schwarzschild GF, where he used isotropic Schwarzschild coordinates. Linet improved the calculations of Copson, first, by transforming them to standard Schwarzschild coordinates, and second, by adding an extra term which describes a specific influence of the gravitational source on the EF lines.

The advantage of the work of Hanni-Ruffini is that they calculated detailed solution for the EF of a charge located close to the Schwarzschild radius of the gravitational source. Their calculations are based on developing the solution in Legender polynomials, and one has to calculate the EF by developing these solutions in increasing powers of l (the polynomial power). Thus, one finds that for close locations of the charge to the GF source, low powers of l are needed, and the EF can be calculated accurately. However, for larger distances of the EF source from the GF source, higher and higher powers of l are needed, and the calculations become impractical. Indeed, in their first paper, Hanni-Rufini carry their calculations for chrages located at distances of few times r_s (Schwarzschild radius) from the GF source. In a recent paper, Ruffini [17], who gives a broad review of this field, mentions the work carried out by Linet.

The potential V of an electric charge located in a Schwarzschild gravitational field at a point $r = a$ is given by Linet [15]:

$$V(r,\theta,\phi) = \frac{q}{ar}\Big[(r-m)(a-m) - m^2\lambda\Big] \times \Big[(r-m)^2 + (a-m)^2 - m^2 - 2(r-m)(a-m)\lambda + m^2\lambda^2\Big]^{-1/2} + \frac{qm}{ar}, \quad (7)$$

where $\lambda = \cos\theta\cos\theta_0 + \sin\theta\sin\theta_0\cos(\phi-\phi_0)$. In expression (7), q is the charge, r is the distance from the coordinates origin which is located at the mass that creates the Schwarzschild field, and m deisgnates the mass, in units of GM/c^2.

This potential can be written in a more convenient form (separating powers of m):

$$\begin{aligned} V(r,\theta,\phi) = \\ & q\Big[1 - m(1/a+1/r) + \frac{m^2}{ar}(1-\lambda)\Big] \\ & \times\Big[r^2 + a^2 - 2ra\lambda - 2m(1-\lambda)(r+a) \\ & + m^2(1-\lambda)^2\Big]^{-1/2} + \frac{qm}{ar} \\ & = \frac{q}{x}\Big[1 - \frac{m}{ar}\big[a + r - m(1-\lambda)\big]\Big] \\ & \times\Big[1 - \frac{2m}{x^2}(1-\lambda)(r+a) + \frac{m^2}{x^2}(1-\lambda)^2\Big]^{-1/2} + \frac{qm}{ar}\,. \quad (8) \end{aligned}$$

We want to work in the plane in which $\theta = \theta_0 = \pi/2$, $\sin\theta = 1$, $\cos\theta = 0$, $\phi_0 = 0$, and $\lambda = \cos\phi$. From these definitions it is clear that the expression in the denominator which is free from m, $x = (r^2+a^2-2ra\lambda)^{1/2}$ represents the distance of the field point from the charge, and when $m \to 0$, we recover the regular expression for an electric potential.

This potential can be differentiated with respect to the coordinates $(r,\ \phi)$, thus obtaining the components of the electric field. From these components, one can obtain the equation for the field lines as a function of the coordinates, and by using numerical integration one can plot the field lines of the EF located at certain

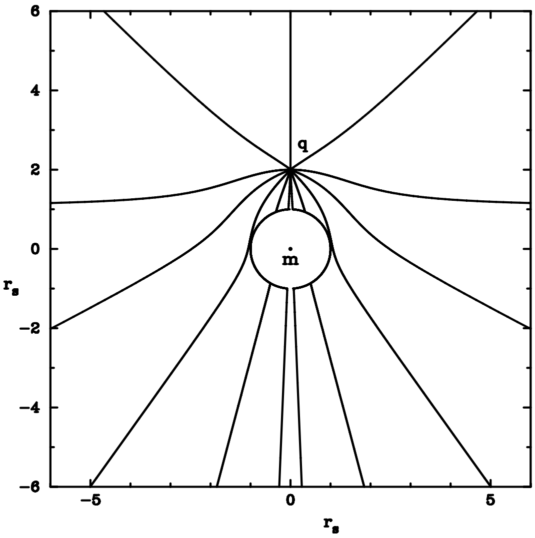

Fig. 4. The field lines of a charge located at a distance $a = 2r_s$ from the mass m.

locations in space. The equation for the field lines as given by Hanni-Ruffini [14] is:

$$\frac{dr}{d\phi} = \left(1 - \frac{2m}{r}\right) r^2 \frac{(\partial V/\partial r)}{(\partial V/\partial \phi)}. \tag{9}$$

We start the numerical integration from the charge in a certain direction, and using eq. 9 we proceed, where at each integration step we calculate the derivatives of V with respect to r and ϕ. At each step the inegration of eq. 9 is carried by using the predictor-corrector method [18], which is the least expensive integration method that yields very accurate results. In order to compare results with those of Hanni-Ruffini we began with a charge located at a distance of $2r_s$ from the mass (r_s = Schwarzschild radius). The results of this integration are shown in Fig. 4, and they can be compared with those of Hanni-Ruffini ([14] Fig. 3).

The lines of force reach the event horizon (as pointed out in [14]) and follow it up to a certian angle ϕ, at which they leave the event horizon. Inspection of eq. 9 gives the mathematical explanation for this behaviour of the field lines: We observe that at the event horizon ($r = r_s = 2m$), $dr/d\phi = 0$. The distance from the central mass remains constant, and the line of force follows the surface of the black hole, until it reaches a point at which also $dV/d\phi = 0$.

At this point the line of force leaves the black hole surface. At the critical angle (the third line from the center in Fig. 4) the line of force just grazes r_s, and for larger angles the lines do not touch r_s. However, all lines of force are influenced by the presence of the central mass.

In Fig. 5 we present similar maps of the field lines from charges located in a Schwarzschild field at larger distances from the central mass, $a = 10r_s$.

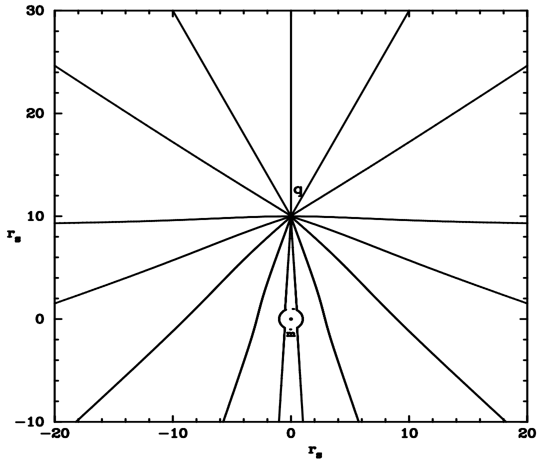

Fig. 5. The field lines of a charge located at a distance $a = 10r_s$.

From Fig. 5 we learn that when the charge is located at distances greater than few tens of r_s from the central mass, most of the field lines are hardly influenced by the presence of the mass, except from those that pass very close to the black hole surface.

These calculations can be used to analyze possible effects of gravity on charges moving close to the surface of black holes, like the calculations carried by Bekenstein [19].

5. Maxwell equations

A basic demand on any electrical phenomenon is that it should satisfy Maxwell equations. For the case of an accelerated charge, this demand is satisfied automatically, where all the physical phenomena involved, including the creation of radiation by the accelerated charge can be driven directly from Maxwell equations (see [4, 5]).

However, for the case of a charge supported statically in a gravitational field, the role of Maxwell equations is not clear at a first glance. The charge is static and it certainly does not supply any dynamical effect. (This fact drives the instinctive conclusion that in this situation no radiation is created). However, we should consider the fall of the electric field in the gravitational field and derive the dynamical effects of this fall.

The idea that an EF is an independent physical entity and should be treated accordingly, is not accepted intuitively by all members of the physics community. However, Einstein expresses this approach very clearly [1], and it is also supported by Landau and Lifshitz [2]. The important property that emerges from Einstein's approach, and is also demonstrated by the way in which the field of a moving charge is calculated by the retarded potentials method is, that an EF is continuously induced by a charge on space, and this field expands in free space with a constant velocity, c. Since this field possesses mass (energy) density, it is subject to gravity and it falls in a GF. This is a dynamical process whose effects should be studied accordingly.

In order to calculate this dynamical effect we use Maxwell equation:

$$\nabla \times \mathbf{B} = \frac{1}{c}\frac{d\mathbf{E}}{dt} + \frac{4\pi \mathbf{J}}{c}, \qquad (10)$$

and the vector identity:

$$\nabla \cdot (\mathbf{E} \times \mathbf{B}) = \mathbf{B} \cdot (\nabla \times \mathbf{E}) - \mathbf{E} \cdot (\nabla \times \mathbf{B}). \qquad (11)$$

Since $\mathbf{J}$ and $\nabla \times \mathbf{E}$ vanish in the case of a static charge in a gravitational field, we are left with:

$$\nabla \cdot (\mathbf{E} \times \mathbf{B}) = -\mathbf{E} \cdot (\nabla \times \mathbf{B}) = -\frac{1}{c}\, \mathbf{E}\frac{d\mathbf{E}}{dt}. \qquad (12)$$

No motion of the charge is involved, but the motion of the EF, which falls in the GF, **is** involved. Due to this fall, $d\mathbf{E}/dt$ does not vanish. There is a source for the energy flux of the radiation, which is given by: $\mathbf{E} \times \mathbf{B}$.

More precisely, we can write:

$$\frac{1}{4\pi}\nabla \cdot (\mathbf{E} \times \mathbf{B}) = -\frac{1}{4\pi c}\, \mathbf{E}\frac{d\mathbf{E}}{dt}$$
$$= -\frac{1}{c}\frac{d}{dt}\left(\frac{\mathbf{E}^2}{8\pi}\right) = -\frac{1}{c}\frac{d}{dt}\Sigma \quad , \qquad (13)$$

where $\Sigma = \mathbf{E}^2/8\pi$ represnets the energy density of the electric field. This energy density is subject to gravity, and its fall supplies the dynamical effect that creates radiation. From eq. 13 we learn that the flow of the energy density of the EF supplies the energy carried by the radiation.

6. Summary

The electric field of a static charge in different gravitational fields is studied. In all these cases, the electric field of the static charge becomes curved, due to the fall of the electric field in the gravitational field. This configuration is compared to the configuration of the electric field of an accelerated charge in free space, where the field equations are calculated by the use of STR formalism. This curvature creates a stress force, which, when interacting with the static charge "tries" to brake the fall of the electric field from being a free fall. In order to keep the the free fall of the electric field, the gravitational source has to perform an extra work to keep the electric field free falling. This extra work is the source of the energy carried by the electrocmagnetic radiation created in this configuration. Maxwell equations are used to calculate the energy flux carried by this radiation, and to relate it quantitavily to the motion of the electric field.

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