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A Brief Note on Dirac's Membrane and p -branes

Abstract

In this brief note, we point out that Dirac's original membrane (or shell) model of the electron and its reincarnation, the p-branes of superstring or M-theory follow from simple but overlooked considerations in the original theory of the electron equation.

In 1962 Dirac introduced a model of the electron which in its simplest terms was a spherical shell [1]. The important features of this model were that the electron had a finite self energy and only two parameters were required, viz., the mass and charge, as in the point particle case. There were excited states describing possibly the spectrum of heavier particles. On the other hand, Dirac's action did not contain the minimum coupling terms between the charge and the electromagnetic field. This coupling was obtained by a boundary condition and was consistent in the special gauge in which the potential on the membrane's surface was zero. Later this model was studied by Barut, Pavsic and others [2,3,4]. In these studies a covariant theory of a moving charge memberane in an arbitrary dimension coupled to the electromagnetic field was considered and developed. Interestingly there has been a return to similar ideas in M-theory, which is currently in vogue amongst superstring theorists. In general p-branes are being considered. In M-theory coordinates become matrices and this leads to a noncommutative geometry [5].

In this brief note we would like to point out that the above brane prescription can be obtained in a straightforward manner, originating from the original Dirac theory of the electron itself, something which has been long overlooked including by Dirac himself.

Indeed in the theory of the Dirac equation for the electron [6] we have effectively

$$i\hbar \frac{d}{dt}(u_i) = -2mc^2(u_i), \tag{1}$$

$$i\hbar \frac{d^2}{dt^2}(u_i) = 2mc^2(\dot{u}_i), \tag{2}$$

(Dirac himself used the notation $c\alpha_i$ for u_i).

We would like to point out that these equations imply that the electron is a rotating shell at the Compton wavelength. For in this case we would have (Cf.ref.[7] for details)

$$\left|\frac{du_v}{dt}\right| = |u_v|\,\omega$$

where,

$$\omega = \frac{|u_v|}{R} = \frac{2mc^2}{\hbar}$$

and (2) above. These equations would also follow directly from a noncommutative geometry viz.,

$$[dx^\mu, dx^\nu] \approx \beta^{\mu\nu}\ell^2 \neq 0 \tag{3}$$

if ℓ were at the Compton scale.

On the other hand it has been argued by the author that the noncommutative geometry (3) which is valid if there is a minimum space time cut off at an arbitrary length ℓ, is particularly interesting, when ℓ is the Compton wavelength (Cf.[5,7,8] for a detailed discussion). It is worth pointing out that such a Compton length cut off was argued by Dirac himself in connection with his electron equation [6]. In this case the position coordinate is non Hermitian and the velocity of the electron equals that of light. Dirac pointed out that once averages over the Compton scale are taken, both these unphysical features disappear. (In other words, something which he did not mention, there is a minimum Compton scale cut off leading to (3)).

In conclusion we would like to point out that the ingredients for the shell model and p-branes of M-theory were already present but overlooked in Dirac's original electron theory. Once a noncommutative geometry or equivalently a minimum cut off at the Compton scale is considered as in the author's work (Cf.refs. [5] and [7]), then these results follow.

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