

# Tritium Beta Decay in a Left-Right Symmetric Model

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## Abstract

We start with a Left-Right Symmetric Model and we analyze the endpoint of the beta decay of the tritium  ${}^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e$ . We applied this model to incorporate the right currents, whereby we propose an amplitude whose leptonic part contains the parameter  $\lambda$  defined as a left-right asymmetry parameter which measures the parity violation. We realized the numerical computation to the sensibility of the experiments Mainz and Troitsk of  $m_{\nu_e} = 2.2$  eV. We find that the spectrum of energy of the electrons for this experiments it is not affected by the left-right asymmetry parameter.

## 1. Introduction

In modern particle physics, one of the most intriguing and most challenging tasks is to discover the rest mass of neutrinos which bear fundamental implications for particle physics, astrophysics and cosmology. Until recently, the Standard Model (SM) [1] of particle physics assumed neutrinos to be massless. However, actual investigations of neutrinos from the sun and of neutrinos created in the atmosphere by cosmic rays, in particular the recent results of the Super-Kamiokande experiment on the neutrino oscillations [2] as well as on the GALLEX, SAGE, GNO, HOMESTAKE and Liquid Scintillator Neutrino Detector (LSND) [3] experiments, have given strong evidence for massive neutrinos indicated by neutrino oscillations.

The existence of neutrino oscillations and therefore, of neutrino mixing and masses, has far-reaching implications to numerous fields of particle physics, astrophysics and cosmology. The SM of particle physics, which very precisely describes the present experimental data up to the electroweak scale, offers no explanation for the observed pattern of the fermion

masses or the mixing among the fermion generations. In particular, it offers no explanation for neutrino masses and neutrino mixing. Accordingly, the recent experimental evidence for neutrino masses and mixing is the first indication of physics beyond the Standard Model.

There are many theories beyond the Standard Model which explore the origins of neutrino masses and mixings. In these theories, which often work within the framework of Supersymmetry, neutrinos naturally acquire mass. A large group of models makes use of the so-called see-saw effect to generate neutrino masses [4]. Other types of theories are based on completely different possible origins of neutrino masses such as radiative corrections arising from an extended Higgs sector [5].

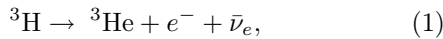
In astrophysics and cosmology, neutrino masses and mixings play an important role in numerous scenarios ranging from the formation of light nuclei during the Big Bang nucleosynthesis and the formation and evolution of large scale structures in the universe, to stellar evolution and the very end of a heavy star, i.e., a supernova explosion [6]. Of special interest are the relic neutrinos left over from the Big Bang. The

number of these neutrinos in the universe is huge, equivalent to the photons of the Cosmic Microwave Background Radiation (CMBR). The ratio of relic neutrinos to baryons is about  $10^9 : 1$ , therefore even small neutrino masses are of great importance.

The best neutrino mass limits have been extracted from measurements of the tritium  $\beta$ -decay spectrum close to its endpoint. Since neutrinos are very light particles, a mass measurement can best be performed in this region of the spectrum as in other parts the nonlinear dependencies caused by the relativistic nature of the kinematic problem cause a significant loss of accuracy. This by far overwhelms the possible gain in statistics one could hope for. Two groups in Mainz and Troitsk used spectrometers based on Magnetic Adiabatic Collimation combined with an Electrostatic filter (MAC-E technique), which obtained the same value

$$m_{\nu_e} < 2.2 \text{ eV} [7, 8], \quad 95 \% C.L.$$

Determination of the absolute scale of neutrino masses is one of the most important and, at the same time, challenging problems in neutrino physics. Currently, the study of the electron energy spectrum near the endpoint of the Tritium beta decay



is the most sensitive direct method of determining the scale of masses. It is well-known that in the absence of mixing, the energy spectrum of the emitted  $e^-$  is described by

$$\frac{d\Gamma}{dE_e} = \frac{G_F^2}{2\pi^3} |\mathcal{M}|^2 F(Z, R_e, E_e) \times p_e E_e (E_e^{max} - E_e) \sqrt{(E_e^{max} - E_e)^2 - m_\nu^2}, \quad (2)$$

where  $G_F$  is the Fermi constant,  $p_e$ ,  $E_e$  and  $E_e^{max}$  are the momentum, energy, and maximum endpoint energy, respectively, of the electron, and  $|\mathcal{M}|^2$  is the absolute square of the nuclear matrix element. The Fermi function,  $F(Z, R_e, E_e)$ , captures the correction due to the Coulomb interactions of the electron with the charge  $Ze$  of the daughter nucleus [9]. We adopt the usual expression [10], derived from the solutions of the Dirac equations for the point-nucleus potential  $-Z\alpha/r$  evaluated at the nuclear radius  $R_e$  [11]. As both  $\mathcal{M}$  and  $F(Z, R_e, E_e)$  are independent of  $m_\nu$ , the dependence of the spectral shape on  $m_\nu$  is given by the phase space factor only. In addition, the bound on the neutrino mass from tritium  $\beta$  decay is independent of whether the electron neutrino is a Majorana or a Dirac particle.

The investigation of this decay has several advantages. Since tritium beta decay is a super-allowed transition, the nuclear matrix element is a constant and the electron spectrum is determined by the phase space.

There are many extensions of the standard model that predict measured effects of deviations of the standard model in decays at tree body. One of the most popular is the left-right symmetry model, with the norm group  $SU(2)_L \times SU(2)_R \times U(1)$  and currents charged with right helicity [12–14].

The purpose of this paper is to carry out an analysis of the tritium beta decay in the context of a model with left-right symmetry [15]. We start from an extension of the electroweak model applied to the baryons decay [16]. This model contains the parameter  $\lambda$  defined as the parameter of left-right asymmetry which measures the parity violation. We apply this theory to incorporate the right currents, for which we propose an amplitude whose leptonic part is  $V + \lambda A$ , with  $\lambda = -1$  for left currents and  $\lambda = 1$  for right currents. The analysis consists of seeing if the endpoint of the tritium beta decay for  $m_{\nu_e} = 2.2 \text{ eV}$  (Mainz and Troitsk) is affected by the left-right asymmetry parameter.

The signature of an electron neutrino with a mass of  $m_{\nu_e} = 2.2 \text{ eV}$  (Mainz and Troitsk) is shown in Figs. 1–4 in comparison with the undistorted  $\beta$  spectrum of a massless  $\nu_e$ . The spectral distortion is statistically significant only in a region close to the  $\beta$  endpoint. This is due to the rapidly rising count rate below the endpoint  $d\Gamma/dE_e \propto (E_0 - E_e)^2$ . Therefore, only a very narrow region close to the endpoint  $E_0$  is analyzed.

This paper is organized as follows: In Sec. 2 we present the calculus of the reaction  ${}^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e$ . In Sec. 3 we make the numerical computations. Finally, we summarize our results in Sec. 4.

## 2. Tritium Beta Decay in a Left-Right Symmetric Model

In this section, we calculate the differential decay rate of the tritium beta decay in the context of the left-right symmetric model which is described in Ref. [17].

The expression for the Feynman amplitude  $\mathcal{M}$  of the reaction  ${}^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e$ , is

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} [\bar{u}_p \gamma^\alpha (1 - \rho \gamma_5) u_n] [\bar{u}_e \gamma_\alpha (1 + \lambda \gamma_5) v_\nu]. \quad (3)$$

The coefficient  $\lambda$  that appears in the leptonic part of Eq. (3), is what we call the coefficient of left-right asymmetry, which contains information on the effects of the theory V+A (right currents). This coefficient does not appear in common literature and is introduced to incorporate the theory V-A and part of V+A, and, as already commented, is the starting point of this paper.

After applying some of the trace theorems of the Dirac matrices and of sum and average over the initial and final spin, the square of the matrix elements

becomes

$$\sum_s |\mathcal{M}|^2 = 16G_F^2(1 + \lambda^2)m_n m_p E_e E_\nu \times [(1 + 3\rho^2) + (1 - \rho^2)\beta_e \beta_\nu \cos \theta_{e\nu}] \quad (4)$$

where  $\beta_e = p_e/E_e$  and  $\beta_\nu = p_\nu/E_\nu$ .

Our next step, now that we know the square of the Eq. (4) transition amplitude, is to calculate the decay rate of  ${}^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e$ .

In order to calculate the differential energy spectrum of electrons in the  $\beta$  decay with a massive neutrino,  $d\Gamma/dE_e$ , we use the expression for the differential rate of decay for a particle that decays in three, which is expressed by [18]:

$$d\Gamma = (2\pi)^4 |\langle {}^3\text{He} e^- \bar{\nu}_e | \tau | {}^3\text{H} \rangle|^2 \times \frac{1}{2E_{3\text{H}}} \delta^4(p_{3\text{H}} - p_{3\text{He}} - p_{e^-} - p_{\bar{\nu}_e}) \times \frac{d^3\mathbf{p}_{3\text{He}} d^3\mathbf{p}_{e^-} d^3\mathbf{p}_{\bar{\nu}_e}}{(2\pi)^3 2E_{3\text{He}} (2\pi)^3 2E_{e^-} (2\pi)^3 2E_{\bar{\nu}_e}}, \quad (5)$$

where  $|{}^3\text{H}\rangle$  is the initial state of the system,  $\langle {}^3\text{He} e^- \bar{\nu}_e |$  is the final state and  $\tau$  is the operator that makes the transition. For our case,

$$\langle {}^3\text{He} e^- \bar{\nu}_e | \tau | {}^3\text{H} \rangle = \mathcal{M}.$$

The spectrum  $d\Gamma/dE_e$  may be obtained by means of the formulae (5), and after elementary integration with respect to  $d^3\mathbf{p}_e d^3\mathbf{p}_\nu d^3\mathbf{p}_p$ :

$$\frac{d\Gamma}{dE_e} = \frac{G^2}{2\pi^3} (1 + \lambda^2) (1 + 3\rho^2) F(Z, R, E_e) \times p_e E_e (E_0 - E_e) \sqrt{(E_0 - E_e)^2 - m_{\nu_e}^2}, \quad (6)$$

where we have also put in a factor  $F(Z, R, E_e)$ . This factor, called the Coulomb correction factor, appears because of the electrostatic interaction between the charged nucleus and the escaping electron, which modifies the outgoing electron energy. This factor depends on the nuclear charge  $Z$ , the nuclear radius  $R$ , and the electron energy.

### 3. Results

In this section we present our results and conclusions of the beta decay of the tritium. We realized the numerical computation to the sensibility of the experiments Mainz and Troitsk of  $m_{\nu_e} = 2.2$  eV. An important point here is to see the influence of the parameter of left-right asymmetry  $\lambda$  in the spectrum or distribution of energy of the electrons, therefore, a group of figures for different values of this parameter is presented.

We have calculated differential electron energy spectra,  $d\Gamma/dE_e$ , in tritium decay

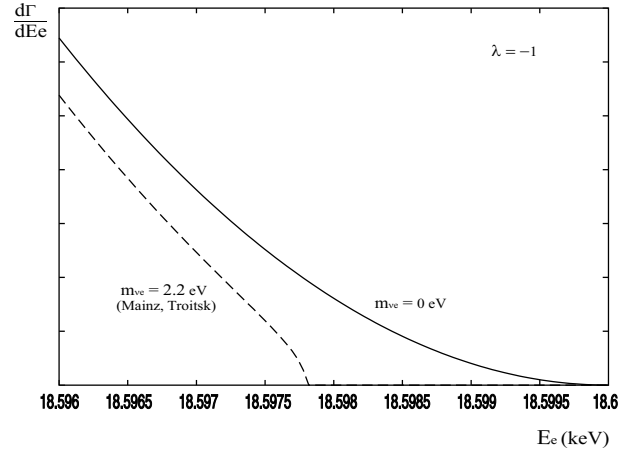


Fig. 1. The electron energy spectrum of tritium  $\beta$  decay. The  $\beta$  spectrum is shown for neutrino masses of 0 and 2.2 eV (Mainz and Troitsk), with  $\lambda = -1$ .

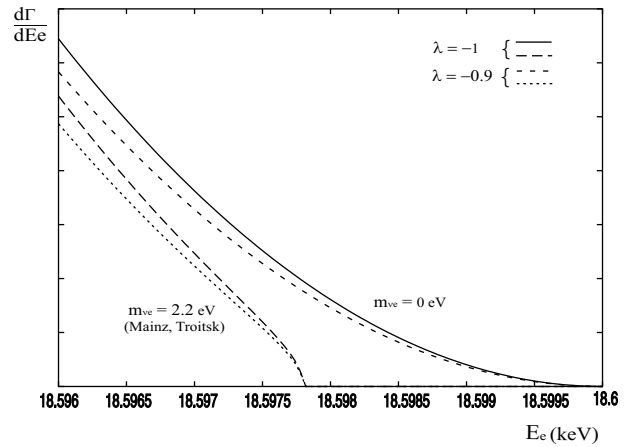


Fig. 2. The same as in Fig. 1, but for  $\lambda = -1, -0.9$ .

using the following values for the mass of  ${}^3\text{H}$  and  ${}^3\text{He}$ :  $M_{3\text{H}} = 2809.4319$  MeV [19] and  $M_{3\text{He}} = 2808.9023$  MeV [19], respectively. The corresponding value for the electron endpoint kinetic energy is  $T_{max} = 18587.56$  eV. Differential electron energy spectra,  $d\Gamma/dE_e$ , corresponding to decays with a massive and massless neutrino in the vicinity of the endpoint, are shown in Fig. 1. This figure corresponds to  $\lambda = -1$ , that is, for left currents.

If  $m_{\nu_e} = 0$ , Eq. (6) immediately shows that  $d\Gamma/dE_e \propto (E_0 - E_e)^2$ . Thus, if we plot the quantity  $d\Gamma/dE_e$  vs  $E_e$ , we would obtain an almost straight line. Such a plot is called the Kurie plot, as shown in Fig. 1 by the solid line. Since the quantity  $d\Gamma/dE_e$  must be non-negative, the maximum value of the electron kinetic energy in this case is  $E_0$ .

However, the almost linearity of the Kurie plot is lost if the neutrino has a non-zero mass (Mainz and Troitsk). The effect of this mass becomes appreciable only near the endpoint of the plot where  $(E_0 - E_e)$

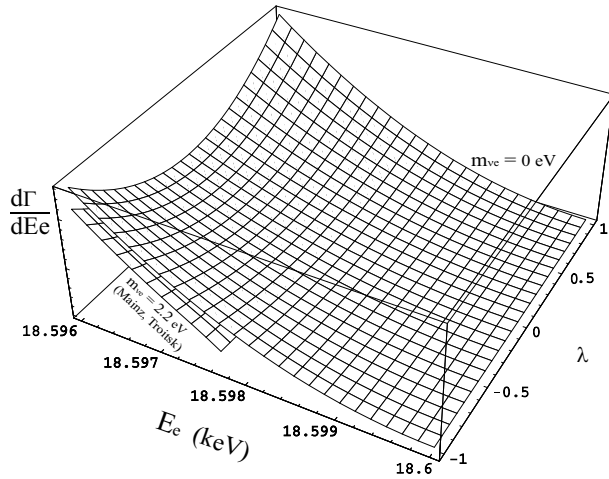


Fig. 3. The differential electron energy spectra for  ${}^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e$  as a function of  $E_e$  and  $\lambda$  for  $m_{\nu_e} = 0$  eV and  $m_{\nu_e} = 2.2$  eV (Mainz and Troitsk).

is comparable to  $m_{\nu_e}$ . Notice that in this case, the maximum kinetic energy of the electron is not given by  $E_0$ , but rather by the vanishing of the quantity inside the square root sign in Eq. (6), i.e.,  $E_e^{max} = E_0 - m_{\nu_e}$ .

Fig. 2 corresponds to  $\lambda = -1, -0.9$ , that is, for left currents and mixture of currents, with  $m_{\nu_e} = 0$  eV and  $m_{\nu_e} = 2.2$  eV (Mainz and Troitsk). From this figure it is clear that the parameter of asymmetry  $\lambda$  does not modify the curve of energy distribution, but only raises or drops it, depending on the value of  $\lambda$ .

Fig. 3 shows the spectrum of energy as a function of the electron energy  $E_e$  for the interval of the parameter of asymmetry  $\lambda$  of  $-1 \leq \lambda \leq -0.9$ , and for the case of  $m_{\nu_e} = 0$  eV and  $m_{\nu_e} = 2.2$  eV. We observe in Fig. 3 that for  $\lambda = -1$ , the data previously reported in the literature is reproduced. Also, for  $m_{\nu_e} = 0$  eV, we observe that the surface is distorted.

In Fig. 4 we plot the differential electron energy spectra,  $d\Gamma/dE_e$ , as a function of the electron energy versus the electron neutrino mass ( $m_{\nu_e}$ ), for  $\lambda = -1$ . We observe in Fig. 4 that for  $m_{\nu_e} = 0$  eV and  $m_{\nu_e} = 2.2$  eV, we reproduce the data previously reported in the literature.

## 4. Conclusions

We have done an analysis on the electron energy spectrum for different values of the parameter of asymmetry  $\lambda$ . The analysis is for  $m_{\nu_e} = 0$  eV,  $m_{\nu_e} = 2.2$  eV (Mainz and Troitsk). The difference between an electron neutrino with mass and the undistorted  $\beta$  spectrum of a massless  $\nu_e$  is clearly observed. The spectral distortion is only significant in a region close to the  $\beta$  endpoint.

In summary, we conclude that the parameter of asymmetry  $\lambda$  does not modify the curve of the tritium

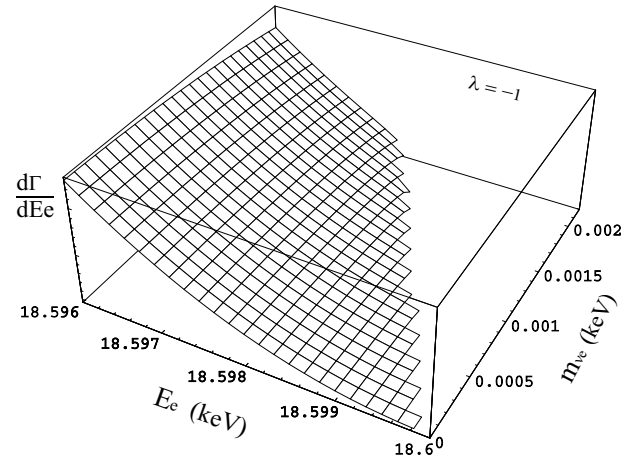


Fig. 4. The differential electron energy spectra for  ${}^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e$  as a function of  $E_e$  and  $m_{\nu_e}$ .

$\beta$  spectrum, but only raises or drops it depending on the value of  $\lambda$ . In the case of  $\lambda = -1$ , we reproduce the curve of the spectrum of energy previously reported in the literature.

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