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Niederhauser's Model for Epilepsy and Wavelet Methods

Contents

Abstract

Wavelets and wavelet transforms (WT) could be a very useful tool to analyze electroencephalogram (EEG) signals. To illustrate the WT method we make use of a simple electric circuit model introduced by Niederhauser [1], which is used to produce EEG-like signals, particularly during an epileptic seizure. The original model is modified to resemble the 10–20 derivation of the EEG measurements. WT is used to study the main features of these signals.

1. Brain and Neurons

The body of animals, including the human being, is controlled by the nervous system. This system has a primary division: central and peripheral. The brain, or cerebrum, the cerebelum, and the spinal cord form the central nervous system, while the peripheral structure is integrated by long nerves that reach every part of the body. The brain is organized in zones which perform specific tasks, which nowadays are the subject of more detailed studies.

At the microscopic scale, the basic functional units of the brain and nerves are a class of excitable cells called neurons. The human brain alone contains about 10^{11} neurons which come in different shapes and sizes but have the same general morphology. The soma (body) of a neuron can measure from 1 μm up to 1 mm across, contains the nucleus, and has two main sets of membrane elongations:

(i) dendrites are prolongations through which the neuron receives information from other neurons

(ii) the axon is the main prolongation through which a neuron sends signals to the outside.

2. Brain Cortex

A well-defined spatial organization of the human brain is through stacks of layers. The outermost layer is the cortex in which many of the higher activities are performed: memory, attention, perceptual consciousness, thought, and language. This layer is about 3 mm only, but despite the small dimension is of basic interest in the research of neuro-physiologists

Fig. 1. This is a photomicrograph from Cajal's preparations (housed in the Museo Cajal at the Cajal Institute, Madrid, Spain) of a neuron from the cerebral cortex of a newborn infant, impregnated by the Golgi stain. The soma, the axon, and the dendrites are easy to identify. Courtesy of http://nobelprize.org/medicine/articles/cajal/

because it engenders important features of the human thought. The cortex is usually studied through the electric signals that produces. The way to detect these signals is through the electroencephalogram (EEG), which is basically a record of the electric activity as obtained by electrodes on the scalp. The EEG could be understood as a superposition of the individual signals coming from each neuron in a given lapse of time [2]. This makes it a very useful tool as an experimental surface measure of the activity of a certain number of neurons that are of specific interest [3].

2.1. Basics of EEG

The history of EEG begins in 1875 when Richard Caton (1842–1926) in Liverpool discovered the existence of electrical signals from the exposed brain of rabbits and monkeys. This discovery was done by employing the galvanometer invented seventeen years earlier by Lord Kelvin. Later, in 1913, the Russian physiologist V. V. Pradvich-Neminsky published the first EEG ever recorded from a dog. At the present time, the EEG is one of the most important methods for the study of neural activity at the level of the brain cortex. It is usually a tool for diagnosis of several important disorders such as autism, language problems, and epilepsy, as well as motor damages.

To obtain EEG data, electrodes should be positioned onto the scalp of the patient. The distribution of the electrodes along with the reference used to measure the signal is called derivation. Though

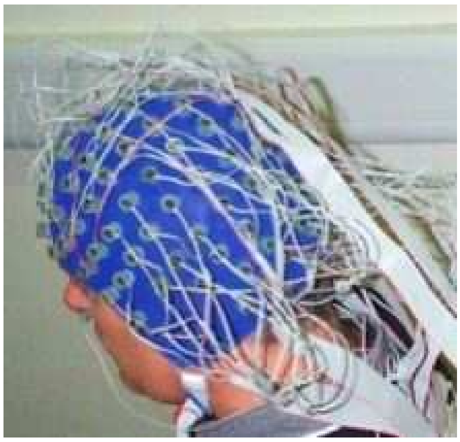

Fig. 2. The 10–20 configuration is the most common to obtain EEG data for diagnosis. The number of electrodes depends on the equipment available and the required precision.

there are several types of derivations, the most commonly used is the 10–20 one. Its name comes form the fact that the electrode arrangement is referred to proportions of skull measures (10 %, 20 % and so on).

3. Niederhauser's Model of Epilepsy

In the original setup of the EEG, the signal is sent from the scalp to moving needles which record it on a sheet of paper. Nowadays, experts make use of samplers and computers to create data sets to represent the EEG as a set of channels which resemble the usual EEG. Since the EEG is a set of time series which reflect the activity of different groups of neurons, it is possible to describe the behavior of a cluster of neurons with a few simple interaction rules. This was the basic idea of Niederhauser [1] who proposed a discrete model on which we will focus in the following. The model takes into account basic features of real neurons to produce an EEG like signal at normal periods and also through the so-called epileptic seizures, which roughly means a sudden start of a regime of strong oscillations.

On the other hand, epilepsy is a very complicated disease and has different manifestations. The model proposed by Niederhauser is thus referred solely to epileptic seizures possessing apparent dominant frequencies. These crises are associated to the hypersynchrony of large groups of neurons and some degree of order is considered theoretically. The neuronal units (called neuronions) are distributed within a rectangular zone array with a set of simple interaction rules. The neuronions are programmed to transport electric charge from one zone to another in a conditional way when a threshold charge difference is reached. If the charge difference is below the threshold,

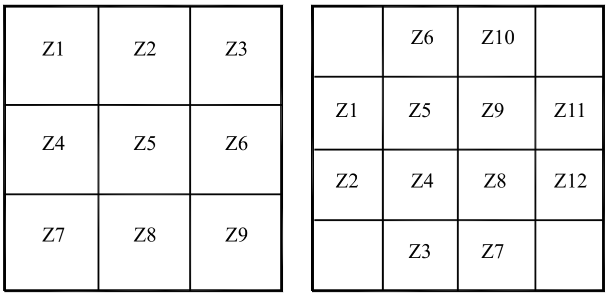

Fig. 3. Left: the original Niederhauser's configuration. Right: the 10-20 configuration.

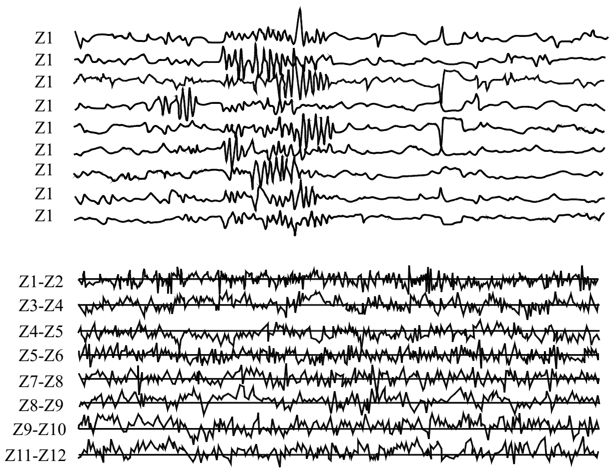

Fig. 4. Output signals generated by our simulations of the original Niederhauser's model (top) and the 10-20 derivation (bottom).

the neuronion has only a small probability to fire. The target zone of each neuron is random and a threshold value for the charge transportation has to be set up at the beginning of the simulation.

The original model considers $2 \cdot 10^4$ neuronions distributed over nine regions in a rectangular 3×3 array as shown in fig. 3. The most important parameter of the model is the threshold voltage which is in direct correspondence with the firing threshold of a real neuron. If a large threshold value is chosen, the output of the simulation will resemble a normal EEG signal, whereas a small threshold value will yield a seizure-like output. To make the original model more realistic, a larger number of neurons were distributed over a 4×4 arrangement shown in fig. 3 that fits better the simplest 10-20 derivation. We found that the output signal did not change significantly (see fig. 4), which means that the configuration of the zones is not a critical parameter in the modeling.

4. Wavelet Theory

Wavelet transforms (WT) are generalized Fourier transforms that in the last two decades have been extensively used to investigate special features of real functions such as scalar one dimensional fields, and more usually, time series. The WT has significant advantages over the common Fourier transforms. The most simple way to argue in favor of the WT is that, unlike the non-localized Fourier spectrum, the WT gives details of the signal at different resolutions and portions of the entire signal.

In general, wavelets are functions in the class $\psi(t) \in L^2(\mathbb{R})$ with the following properties:

$$C_\psi = \int_0^\infty \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < \infty \qquad (1)$$

$$\int_{-\infty}^{\infty} \psi(t)dt = \hat{\psi}(0) = 0, \qquad (2)$$

where $\hat{\psi}(\omega) = \int e^{i\omega t}\psi(t)dt$ is its Fourier transform.

The first equation is an admissibility condition, while the second one is the zero mean condition. The function $\psi(t)$, known as the mother wavelet, can be used to build an orthonormal basis of translated and dilated functions of the form

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right) . \qquad (3)$$

The WT of a function f that we denote by $\hat{f}_{a,b}(t)$ is defined as the scalar product in $L^2(\mathbb{R})$ of the function with the chosen wavelet:

$$\hat{f}_{a,b}(t) = \langle f, \psi_{a,b} \rangle. \qquad (4)$$

The WT measures the variation of f in a neighborhood of size proportional to a centered on point b.

One fundamental property that is required in order to analyze singular behavior is that $\psi(t)$ has enough vanishing moments. A wavelet is said to have n vanishing moments if and only if it satisfies

$$\int_{-\infty}^{\infty} t^k \psi(t)dx \quad = \quad 0, \text{ for } k = 0, 1, \ldots, n-1 \qquad (5)$$

and

$$\int_{-\infty}^{\infty} t^k \psi(t)dt \quad \neq \quad 0, \text{ for } k \geq n. \qquad (6)$$

This means that a wavelet with n vanishing moments is orthogonal to polynomials up to order $n-1$. In fact, the admissibility condition requires at least one vanishing moment. So the wavelet transform of $f(t)$ with a wavelet $\psi(t)$ with n vanishing moments is nothing but a "smoothed version" of the nth derivative of $f(t)$ on various scales. When one is interested to measure the local regularity of a signal this concept is crucial. In the plots of fig. 5 we used Daubechies wavelets with 8 and 20 vanishing moments, respectively.

As the set of wavelets form a basis, any function can be decomposed into the linear combination

$$f(t) = \sum_m \sum_n x_n^m \psi_{m,n}(t), \tag{7}$$

with coefficients

$$x_n^m = \int_{-\infty}^{\infty} f(t)\psi_{m,n}(t)dt, \tag{8}$$

where the basis functions are defined in terms of the mother wavelet as follows

$$\psi_{m,n}(t) = 2^{m/2}\psi(2^m t - n). \tag{9}$$

In the wavelet approach the fractal character of a certain signal can be inferred from the behavior of its power spectrum $P(\omega)$, which is the Fourier transform of the autocovariance (also termed autocorrelation) function and in differential form $P(\omega)d\omega$ represents the contribution to the variance of a signal from frequencies between ω and $\omega + d\omega$.

$$P_\varphi(\omega) \sim |\omega|^{-\gamma_f} \ , \tag{10}$$

where γ_f is the spectral parameter of the wave signal.

Indeed, it is known that for self-similar random processes the spectral behavior of the power spectrum is given by [4, 5]

In addition, the variance of the wavelet coefficients possesses the following behavior [5]

$$\operatorname{var} x_n^m \approx (2^m)^{-\gamma_f} \ . \tag{11}$$

These results will be employed to study the output of the Niederhauser model, and also of real EEG data for comparison purposes.

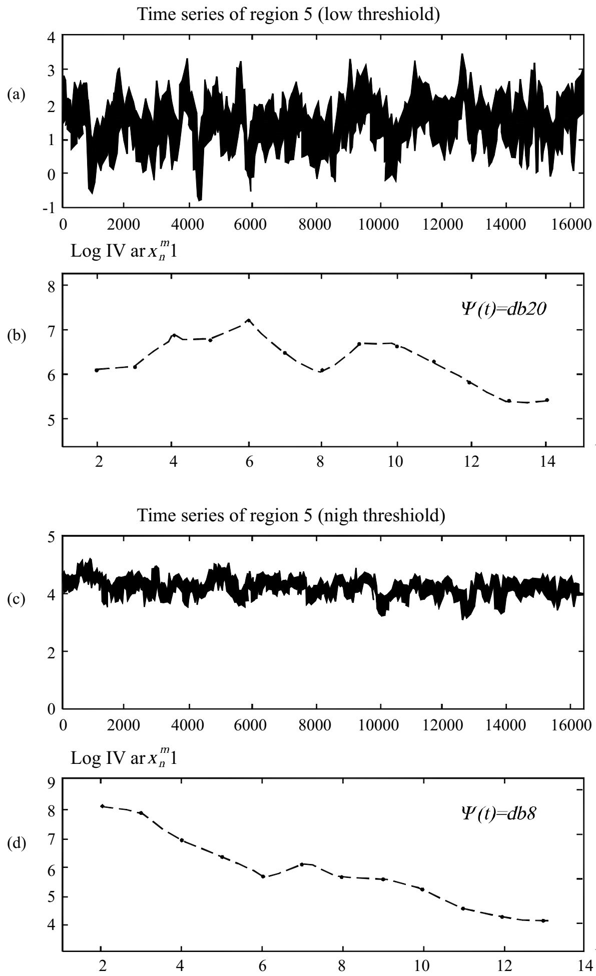

Fig. 5. The time series above correspond to the 5th zone of an original Niederhauser's arrangement. Plots (a) and (b): an epileptic seizure event simulation is shown for which the wavelet coefficients do not display a self-similar fractal structure. Plots (c) and (d): a normal EEG simulation is shown for which the wavelet coefficients could be argued to have a fractal behaviour.

5. WT and EEG Signals

It is relatively easy to use wavelet theory to analyze EEG data, although the interpretation of the results is not so easy. There is previous work that links the time series analysis through the WT to the analysis of EEG data. The detection of the so called epileptic spikes is explained in [6], where the authors also mention a comparison between this method and the available software within the medical community. In this work, wavelet theory will be applied to the model by Niederhauser in the particular case of epilepsy.

The normal EEG is sometimes thought of as a chaotic signal. There is some discussion about this issue in [7] and previous works, where Lyapunov exponents theory is used to measure chaos. Wavelet analysis provides a simple algorithm to determine the fractal dimension (closely related to the Hausdorff-Besicovich dimension) of a curve, and therefore conclude whether it is a fractal or not.

In fig. 5 simulations of the EEG with the original Niederhauser's model are shown. The wavelet coefficients reveal a fractal behaviour in the normal EEG while in the epileptic seizure, the coefficients cannot give us such information. We find that during the epileptic seizure there is a dominance of a given scale, which could be interpreted as an ordering of neurons at a certain scale.

The behaviour of the output of the modified system is qualitatively the same. This means that the fractal and nonfractal feature of the respective episodes are constant. From these computations one could conclude that the normal EEG has a fractal feature. Despite these results, the same analysis for real EEG is missing, though the same results are expected. Additionally, the results of the simulations support the idea that at an epileptic seizure there is some degree of order in the EEG signals.

6. Conclusions

We reconsidered the simple electric circuit model of Niederhauser for epilepsy with minor modifications. We confirm that it is capable to reproduce specific features of EEG data such as frequency or scale dominance at a seizure and fractality at normal periods. This model is useful for checking different methods for EEG signal analysis and gives insight to non-medic students on certain basic features of epilepsy. It could even give a clue of the causes and behaviour of the disease itself if appropriate modifications are performed. As an example, we used wavelet transform analysis since we believe it could be a useful tool in getting a wealth of information about particular features of the EEG signals from pathological conditions in different patients to specific details about a given patient. In the future, we hope to make further modifications of the model and the analysis of the data through wavelet analysis to seek for more details of epileptic disorders and their relationships to neuronal dynamical features at the level of the whole brain. A software development with characteristics similar to current software, such as spike detection, through wavelet transform is under consideration.

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