PACS №: 41.20.Jb; 03.50.De; 41.60.-m

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Waves Produced by Sources on an Expanding and Moving Disk

Contents

Abstract

Solutions of the inhomogeneous wave equation for sources on an expanding and moving disk are discussed. An algorithm for constructing these solutions in terms of modes of the cylindrical coordinate system is given. We demonstrate how to use them in the special case where sources move with the velocity of wavefront and expand with a velocity that is less, equal or greater than the wavefront velocity. A family of "superluminal" solutions is obtained. The possibility of description of electromagnetic waves by means of the scalar solutions is shown.

1. Introduction

In the recent paper [1] the solutions of the inhomogeneous wave equation were constructed for sources distributed on a circular frame at rest expanding with a velocity that is less, equal or greater than the wavefront velocity. There are some reasons to expect that these results can be extended to more general problems of wave formation by sources on an expanding disk that starts motion at fixed moment of time and travels along a straight line with an arbitrary constant velocity. This report presents an attempt to construct the transient solutions describing wave perturbations produced by the above-mentioned sources. First, we give an algorithm for constructing solutions of the initial-value problem in terms of modes of the cylindrical coordinate system, that is, as a Fourier series, and use the obtained results for description of the wave functions formed by some sources. After that, using special scalar potentials, we discuss a possibility to calculate the components of the electromagnetic field. So, we give the representation of the components for the radial current distributed on the circular frame in terms of modes of the cylindrical coordinate system. Previously obtained analogous expansions were constructed for circular sources on a spherical surface expanding with the wavefront velocity [2]. The present research is connected with the problems launching directed scalar and electromagnetic "subluminal-superluminal" waves and distortion of the signal shape [3,4], as well as with investigations of elaboration of non-traditional radiators.

2. Basic relations

We write the source function j in cylindrical coordinates ρ, φ, z as

$$j = \frac{1}{2\pi}\,\delta(z-\beta\tau)h(\varepsilon\tau-\rho)f(\rho,\varphi,z,\tau),\ \tau>0, \quad (1)$$

where $\delta(x)$ is the Dirac function, $h(x)$ is the Heaviside function, f is a continuous function, $\tau = ct$ is the time variable, c is the wavefront velocity, $\varepsilon = \mathrm{v}/c \in (0,\infty)$ and $\beta = \upsilon/c \in (0,\infty)$ are constants, v and υ are expansion and motion velocities correspondingly, and we formulate the initial-value problem for the 3D wave equation

$$\left(\partial_\tau^2 - \frac{1}{\rho}\partial_\rho(\rho\partial_\rho) - \frac{1}{\rho^2}\,\partial_\varphi^2 - \partial_z^2\right)\psi = \frac{4\pi}{c}\,j,$$
$$\psi = j \equiv 0, \quad \tau<0. \quad (2)$$

Constructing solutions of the problem, we apply the method similar to that used in [1]. Representing the wavefunction ψ and the source function j in terms of the Fourier series

$$\psi = \sum_m e^{im\varphi}\psi_m(\rho,z,\tau),$$
$$j = \sum_m e^{im\varphi} j_m(\rho,z,\tau) \quad (3)$$

and using the Fourier-Bessel transformation

$$F_m(s) = \int_0^\infty d\rho\rho J_m(s\rho)F_m(\rho),$$
$$F_m(\rho) = \int_0^\infty dss J_m(s\rho)F_m(s), \quad (4)$$

where $J_m(s\rho)$ is the Bessel function of order m, we obtain the problem for the 1D telegraph equation

$$\left(\partial_\tau^2 - \partial_z^2 + s^2\right)\psi_m(s) = \frac{4\pi}{c} j_m(s),$$
$$\psi_m(s) = j_m(s) \equiv 0, \quad \tau<0, \quad (5)$$

where the right-hand side is

$$\frac{2}{c}\,j_m(s) = \frac{2}{c}\,\delta(z-\beta\tau)\int_0^{\varepsilon\tau} d\rho\rho J_m(s\rho)f_m(\rho,\beta\tau,\tau). \quad (6)$$

This is the Smirnov method of incomplete separation of variables [5]. Then, applying Riemann's formula and the inverse transformation, we write the series coefficients $\psi_m(\rho,z,\tau)$ as

$$\psi_m = \frac{1}{c}\int_0^\tau d\tau' \int_{\tau'+z-\tau}^{-\tau'+z+\tau} dz'\,\delta(z'-\beta\tau')$$
$$\times \int_0^{\varepsilon\tau'} d\rho'\,\rho' f_m(\rho',\beta\tau',\tau')\,I_m(\rho',\beta\tau',\tau'), \quad (7)$$

where

$$I_m = \int_0^\infty dss J_m(s\rho)J_m(s\rho')$$
$$\times J_0\left(s\sqrt{(\tau-\tau')^2-(z-\beta\tau')^2}\right).$$

The integral containing the product of Bessel's functions can be represented in the form (see [6])

$$I_m = \frac{1}{\pi\rho\rho'}\,\frac{\cos m\theta}{\sin\theta}\,H(\rho',\tau',\beta), \quad (8)$$

where

$$H = \begin{cases} 1, & (\tau-\tau')^2-(z-\beta\tau')^2 \\ & \quad \in \left[(\rho-\rho')^2,\ (\rho+\rho')^2\right], \\ 0, & (\tau-\tau')^2-(z-\beta\tau')^2 \\ & \quad \notin \left[(\rho-\rho')^2,\ (\rho+\rho')^2\right], \end{cases} \quad (9)$$

$$\cos\theta = \frac{1}{2\rho\rho'}\left(\rho^2+\rho'^2+(z-\beta\tau')^2-(\tau-\tau')^2\right),$$
$$\sin\theta = \frac{1}{2\rho\rho'}\sqrt{(\tau-\tau')^2-(\rho-\rho')^2-(z-\beta\tau')^2}$$
$$\times\sqrt{(\rho+\rho')^2+(z-\beta\tau')^2-(\tau-\tau')^2}.$$

The integral (7) is easily transformed into

$$\psi_m = \frac{1}{\pi c\rho}\int_{\Phi_1}^{\Phi_2} d\tau' \int_0^{\varepsilon\tau'} d\rho'\,H(\rho',\tau',\beta)$$
$$\times\frac{\cos m\theta}{\sin\theta} f_m(\rho',\beta\tau',\tau'). \quad (10)$$

For $\beta>1$ the integration limits are

$$\Phi_1 = \begin{cases} \dfrac{\tau-z}{1-\beta}, & \text{if } \beta\tau > \mathrm{z} > \tau, \\ 0, & \text{if } \tau > \mathrm{z}, \end{cases} \quad \Phi_2 = \frac{\tau+z}{1+\beta}, \quad (11)$$

while for $\beta<1$ they are

$$\Phi_1 = 0, \quad \Phi_2 = \begin{cases} \dfrac{\tau-z}{1-\beta}, & \text{if } \tau > \mathrm{z} > \beta\tau, \\ \dfrac{\tau+z}{1+\beta}, & \text{if } \beta\tau > \mathrm{z}. \end{cases} \quad (12)$$

In the special case $\beta=1$ we use

$$\Phi_1 = 0, \quad \Phi_2 = \frac{\tau+z}{2}. \quad (13)$$

It is clear that the actual integration domain on the $\tau',\ \rho'$ — *plane* is defined by the limits of the integrals, as well as by the function $H(\rho',\tau',\beta)$.

Expressions (10)–(13) together with the Fourier series give the algorithm to construct the solution of the problem (1) and (2) in terms of modes of the cylindrical coordinate system for different sources of type (1). This solution is a generalization of the results presented in [1].

3. Discussion of applications

As an example of application of the algorithm obtained, we demonstrate how to use them for description of the transient wave produced by the source disk. The parameter $\beta = 1$ is fixed.

3.1. A source expanding with the wavefront velocity

Let us assume that wave perturbations are formed by a source disk moving along the z-axis and expanding with the wavefront velocity, therefore $\beta = \varepsilon = 1$. Then coefficients ψ_m may be written as

$$\psi_m = \frac{1}{\pi c \rho} \int\limits_0^{(\tau+z)/2} d\tau' \int\limits_0^{\tau'} d\rho' \, H(\rho', \tau', 1) \times \frac{\cos m\theta}{\sin\theta} f_m(\rho', \tau', \tau'), \quad (14)$$

where the integrant is defined by the expressions

$$H = \begin{cases} 1, & \tau^2 - z^2 - 2(\tau - z)\tau' \\ & \quad \in \left[(\rho-\rho')^2, (\rho+\rho')^2\right], \\ 0, & \tau^2 - z^2 - 2(\tau - z)\tau' \\ & \quad \notin \left[(\rho-\rho')^2, (\rho+\rho')^2\right], \end{cases} \quad (15)$$

$$\cos\theta = \frac{1}{2\rho\rho'} \left[\rho^2 + z^2 - \tau^2 + \rho'^2 + 2\tau'(\tau - z)\right],$$
$$\sin\theta = \frac{1}{2\rho\rho'} \sqrt{\tau^2 + 2(\tau - z)\tau' - (\rho - \rho')^2 - z^2} \times \sqrt{(\rho+\rho')^2 + z^2 - \tau^2 - 2(\tau - z)\tau'}. \quad (16)$$

Hence we find the actual integration limits using the ρ', τ' – *plane* diagrams and the property of the H function (see [1] and also [6]) and obtain the series coefficients for $\tau < r = \sqrt{\rho^2 + z^2}$ in the form

$$\psi_m = \frac{1}{\pi c \rho} \int\limits_{\Phi_-}^{\Phi_+} d\rho' \int\limits_{\rho'}^{\Theta_{up}} d\tau' \, G_m(\rho', \tau'),$$
$$\Phi_+ > \Phi_- > 0, \ \Theta_{up} > \rho', \quad (17)$$

while for $\tau > r$

$$\psi_m = \frac{1}{\pi c \rho} \int\limits_0^{\tilde{\Phi}_+} d\rho' \int\limits_{\Theta_l}^{\Theta_{up}} d\tau' \, G_m(\rho', \tau') + \frac{1}{\pi c \rho} \int\limits_{\tilde{\Phi}_+}^{\Phi_+} d\rho' \int\limits_{\rho'}^{\Theta_{up}} d\tau' \, G_m(\rho', \tau'),$$
$$\Phi_+ > \tilde{\Phi}_+ > 0, \ \Theta_{up} > \Theta_l > 0, \quad (18)$$

where the integrant

$$G_m = f_m(\rho', \tau') \frac{\cos m\theta(\rho', \tau')}{\sin\theta(\rho', \tau')}. \quad (19)$$

The integration limits

$$\Phi_\pm = -(\tau - \rho - z) \pm \sqrt{2(\tau - \rho)(\tau - z)},$$
$$\tilde{\Phi}_\pm = -(\tau + \rho - z) \pm \sqrt{2(\tau + \rho)(\tau - z)} \quad (20)$$

are found as the coordinates of the intersection points on the τ', ρ' – *plane* of the parabolas

$$\tau' = -\frac{1}{2(\tau - z)} (\rho \pm \rho')^2 + \frac{1}{2} (\tau + z) \quad (21)$$

with the straight line $\tau' = \rho'$, while the limits of the internal integrals

$$\Theta_{up} = -\frac{1}{2(\tau - z)} (\rho - \rho')^2 + \frac{1}{2} (\tau + z),$$
$$\Theta_l = -\frac{1}{2(\tau - z)} (\rho + \rho')^2 + \frac{1}{2} (\tau + z) \quad (22)$$

equal the right-hand side of (21).

Using expressions (17) and (18), one can immediately obtain the series coefficients for sources distributed on circular frames. So, replacing $f_m(\rho', \tau')$ by expression $\tilde{f}_m(\tau')\, \delta(\tau' - \rho')/\rho'$, we obtain

$$\psi_m = \frac{1}{\pi c \rho} \int\limits_{\Phi_l}^{\Phi_{up}} d\tau' \, \frac{1}{\tau'} \tilde{f}_m(\tau') \frac{\cos m\theta(\tau')}{\sin\theta(\tau')}, \quad (23)$$

where $\Phi_{up} = \Phi_+$ and $\Phi_l = \Phi_-$ if $\tau < r$, but $\Phi_l = \tilde{\Phi}_+$ in the opposite case $\tau > r$.

We define functions $\cos\theta(\tau')$ and $\sin\theta(\tau')$ from expressions (16) substituting $\rho' = \tau'$. Then, assuming $m = 0$ and writing

$$\sin\theta(\tau') = \frac{1}{2\rho\tau'} \mathcal{R}(\tau'), \quad (24)$$

where

$$\mathcal{R}(\tau') = \left[(\Phi_+ - \tau')(\tau' - \Phi_-)(\tau' - \tilde{\Phi}_+)(\tau' - \tilde{\Phi}_-)\right]^{1/2},$$

we obtain the axisymmetric solution of the wave equation

$$\psi_0 = \frac{2}{\pi c} \int\limits_{\Phi_l}^{\Phi_{up}} d\tau' \, \tilde{f}_0(\tau') \frac{1}{\mathcal{R}(\tau')}, \quad (25)$$

where the inequalities

$$\Phi_+ - \tau' \geq 0, \quad \tau' - \Phi_- \geq 0,$$
$$\tau' - \tilde{\Phi}_+ \geq 0, \quad \tau' - \tilde{\Phi}_- > 0 \quad (26)$$

are true for both $\tau > r$ and $\tau < r$.

Expressions (17),(18) and (23) together with Fourier series (3) yield explicit solutions of the wave equation. The obtained results allow to investigate the space-time structure of the formed waves. So, we single out two cases that are illustrated in figure 1(a):

(i) the observation point lies outside the sphere $r = \tau$, i.e. $r > \tau$ where we use expression (17) and obtain a real solution inside the domain limited in part by the sphere and in part by the plane $\tau - z = 0$ and the cylindrical surface $\tau - \rho = 0$, which are tangential to the sphere,

(ii) in the opposite case $r < \tau$ the observation point lies inside the sphere, and we use expression (18). Since the expanding disk and circular frame both belong to the plane $\tau - z = 0$ we have superluminal sources and the obtained solutions may be treated as "superluminal" ones, though the fronts of the wave perturbations $\tau - z = 0$, $\tau - \rho = 0$ and $\tau - r = 0$ move with the velocity of the wavefront.

3.2. Expansion velocity greater than the wavefront velocity, $\varepsilon > 1$

Let us assume that a disk source expanding with a velocity greater than the wavefront velocity lies in the plane $\tau - z = 0$, i.e. $\varepsilon > 1$ and $\beta = 1$. We start from expression (10) writing the upper limit of the internal integral as $\varepsilon\tau'$ and present the coefficients ψ_m in a form akin to expressions (17) and (18) for $\tau < r$

$$\psi_m = \frac{1}{\pi c\rho} \int\limits_{\Phi_-}^{\Phi_+} d\rho' \int\limits_{\rho'/\varepsilon}^{\Theta_{up}} d\tau' \, G_m(\rho', \tau'),$$

$$\Phi_+ > \Phi_- > 0, \; \Theta_{up} > \frac{\rho'}{\varepsilon}, \quad (27)$$

and for $\tau > r$

$$\psi_m = \frac{1}{\pi c\rho} \int\limits_{0}^{\tilde{\Phi}_+} d\rho' \int\limits_{\Theta_l}^{\Theta_{up}} d\tau' \, G_m(\rho', \tau') + \frac{1}{\pi c\rho} \int\limits_{\tilde{\Phi}_+}^{\Phi_+} d\rho' \int\limits_{\rho'/\varepsilon}^{\Theta_{up}} d\tau' \, G_m(\rho', \tau'),$$

$$\Phi_+ > \tilde{\Phi}_+ > 0, \; \Theta_{up} > \frac{\rho'}{\varepsilon}, \quad (28)$$

where the integrant G_m is defined by expression (19) and limits of the internal integrals Θ_{up} and Θ_l are defined by expressions (22). The integration limits

$$\Phi_\pm(\varepsilon) = \frac{1}{\varepsilon}\Big[-(\tau - z - \varepsilon\rho) \pm \sqrt{(\tau - z)\,(\tau(1+\varepsilon^2) - 2\varepsilon\rho + z(\varepsilon^2 - 1))}\Big] \quad (29)$$

and

$$\tilde{\Phi}_\pm(\varepsilon) = \frac{1}{\varepsilon}\Big[-(\tau - z + \varepsilon\rho) \pm \sqrt{(\tau - z)\,(\tau(1+\varepsilon^2) + 2\varepsilon\rho + z(\varepsilon^2 - 1))}\Big] \quad (30)$$

are found as the coordinates of the intersection points on the τ', ρ' – *plane* of parabolas (21) and the straight line $\rho' = \varepsilon\tau'$. Collecting expressions (27)–(30) and the Fourier series, we obtain the solution which yield the results of the previous subsection in the limiting case $\varepsilon \to 1$. Note that one can immediately write the coefficients ψ_m for a source on the circular frame from (27),(28), replacing the function $f_m(\rho', \tau')$ by $\tilde{f}_m(\tau')\,\delta(\varepsilon\tau' - \rho')/\rho'$

$$\psi_m = \frac{1}{\pi c\varepsilon\rho} \int\limits_{\Phi_l}^{\Phi_{up}} d\rho' \, \frac{1}{\rho'} \, \tilde{f}_m\left(\rho', \frac{\rho'}{\varepsilon}\right) \frac{\cos m\theta(\varepsilon, \rho')}{\sin\theta(\varepsilon, \rho')}, \quad (31)$$

where

$$\cos\theta(\varepsilon, \rho') = \frac{1}{2\rho\rho'} \times \left(\rho^2 + z^2 - \tau^2 + \rho'^2 + 2\frac{\rho'}{\varepsilon}(\tau - z)\right),$$
$$\sin\theta(\varepsilon, \rho') = \frac{1}{2\rho\rho'} \times \sqrt{\tau^2 + 2\frac{\rho'}{\varepsilon}(\tau - z) - (\rho - \rho')^2 - z^2} \times \sqrt{(\rho + \rho')^2 + z^2 - \tau^2 - 2\frac{\rho'}{\varepsilon}(\tau - z)}. \quad (32)$$

Expressions (29),(30) define integration limits $\Phi_{up} = \Phi_+(\varepsilon)$ and $\Phi_l = \Phi_-(\varepsilon)$ if $\tau < r$, or limits $\Phi_{up} = \Phi_+(\varepsilon)$ and $\Phi_l = \tilde{\Phi}_+(\varepsilon)$ if $\tau > r$.

One can see that the above solutions exist: (i) in the case $\tau < r$ inside the space-time domain limited in part by the sphere $\tau = r$, and in part by the plane $\tau - z = 0$ and the circular cone

$$z = 2\frac{\varepsilon}{\varepsilon^2 - 1}\rho - \frac{\varepsilon^2 + 1}{\varepsilon^2 - 1}\tau \quad (33)$$

with the vertex at the point

$$\rho = 0, \quad z = -\tau\frac{\varepsilon^2 + 1}{\varepsilon^2 - 1} < 0$$

and (ii) in the opposite case $\tau > r$ lies inside the spherical surface $\tau = r$ (see figure 1(b)). The fronts of the wave perturbation move with the wavefront velocity, however the obtained solution may be treated as a "superluminal" one.

3.3. Solutions for $0 < \varepsilon < 1$

It is easy to verify that the expressions from the previous subsection can also be formally used in the

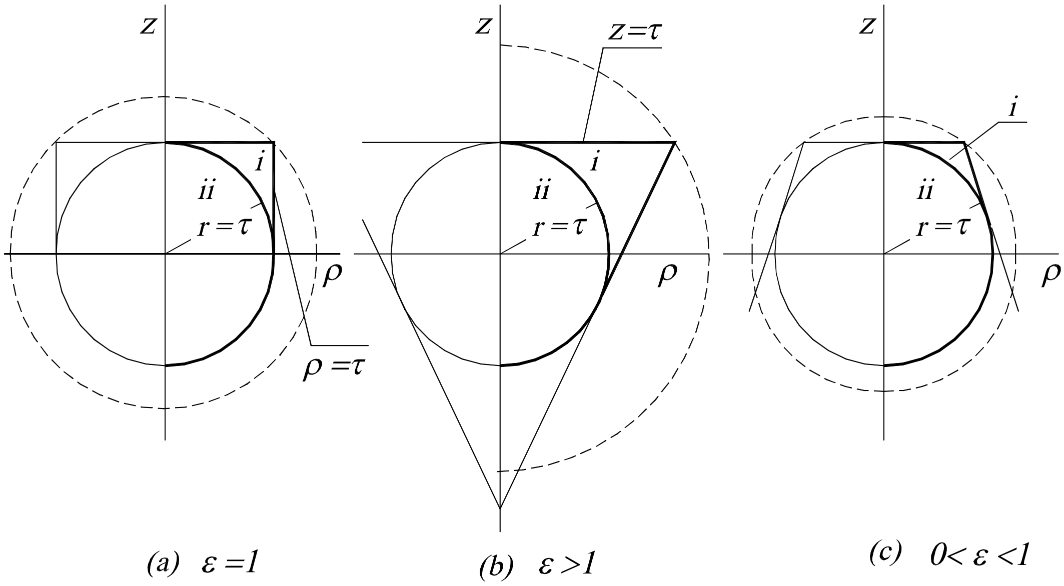

Fig. 1.

case $\beta = 1$ and $0 < \varepsilon < 1$. This fact together with Fourier series (3) immediately yields the last solution of the discussed type. Here we apply the coefficients ψ_m in form (27) if $\tau < r$, and in the form (28) if $\tau > r$. Expressions (29) and (30) are used for the integration limits $\Phi_\pm(\varepsilon)$ and $\tilde{\Phi}_+$ correspondingly, and expressions (22) for Θ_{up} and Θ_l.

The space-time domain where the solution exists is shown in figure 1(c). Note that circular cone equation (31) is also formally true, but the z-coordinate of the cone vertex is

$$z = -\tau(\varepsilon^2 + 1)/(\varepsilon^2 - 1) > 0.$$

Here the wavefront structure is different from the result obtained in subsection 3.2 (see figure 1(b,c)), but the solution may be interpreted as a "superluminal" one.

4. Application to solutions of the Maxwell equations

The obtained solutions of the wave equation can be applied to electromagnetic waves. Below we use SI units and denote the electric induction and magnetic field strength vectors as $\vec{D}$ and $\vec{H}$. There are several ways to describe the electromagnetic field with the help of the scalar functions. So, the Maxwell equations are satisfied for the special case of the electric current density vector $\vec{j} = \vec{e}_z j_z$ (TM waves) if we use

$$\begin{aligned} D_\rho &= \partial^2_{\rho z} u, & H_\rho &= \frac{c}{\rho}\,\partial^2_{\varphi\tau} u, \\ D_\varphi &= \frac{1}{\rho}\,\partial^2_{z\varphi} u, & H_\varphi &= -c\partial^2_{\rho\tau} u, \\ D_z &= (\partial^2_z - \partial^2_\tau) u, & H_z &= 0 \end{aligned} \tag{34}$$

and the function $\psi = \partial u/\partial\tau$ is a solution of the wave equation (2), where j_z is the source function. Here one can treat the function u as the Whittaker-Bromwich potential [7]. Hence we immediately obtain the components of the vector $\vec{H}$

$$H_\rho = \frac{c}{\rho}\,\partial_\varphi\,\psi, \quad H_\varphi = -c\partial_\rho\,\psi. \tag{35}$$

To calculate the components of the vector $\vec{D}$, integration of the relation $\psi = \partial u/\partial\tau$ with respect to time should be performed.

When the current density vector has the radial component only, i.e. $\vec{j} = \vec{e}_r j_r$ (TM waves), one can use the spherical coordinates r, ϑ, φ and express the electric and magnetic field vectors using one scalar function $\tilde{u}$ introduced by Debye and Bromwich [7]

$$\begin{aligned} D_r &= \left(\partial^2_r - \partial^2_\tau\right)\tilde{u}, & H_r &= 0, \\ D_\vartheta &= \frac{1}{r}\,\partial^2_{\vartheta r}\tilde{u}, & H_\vartheta &= \frac{c}{r\sin\vartheta}\,\partial^2_{\varphi\tau}\tilde{u}, \\ D_\varphi &= \frac{1}{r\sin\vartheta}\,\partial^2_{\varphi r}\tilde{u}, & H_\varphi &= -\frac{c}{r}\,\partial^2_{\vartheta\tau}\tilde{u} \end{aligned} \tag{36}$$

that reduce the vector problem to the scalar one. Here the function

$$\frac{\partial\tilde{u}}{\partial\tau} = r\psi \tag{37}$$

is defined from the solution of the wave equation

$$\left(\partial^2_\tau - \Delta\right)\psi = \frac{1}{cr}\,j_r. \tag{38}$$

Expressions (36) together with the solutions of equation (38) allow to represent the components of the electromagnetic field vectors for the radial current distributed on the disk or the circular frame in terms of modes of the cylindrical coordinate system, i.e. as a Fourier series.

As an example, let the current source distributed on the circular frame be formed as an intersection of the conical surface $\vartheta = \vartheta_0$, $\vartheta_0 \in (0, \pi/2)$, and some fictitious sphere expanding with a velocity greater than the velocity of light (see dash-line in figure 1). The expansion starting point and the cone vertex coincide.

Writing the current source function

$$j_r = \frac{1}{2\pi r^2}\, \tilde{f}(r, \varphi, \tau)\, \delta(r - \zeta\tau)\, \delta(\cos\vartheta - \cos\vartheta_0), \quad \zeta = \sqrt{1+\varepsilon^2},$$

in the cylindrical coordinates as

$$j_r = \frac{1}{2\pi\rho}\, \delta(\rho - \varepsilon\tau)\, \delta(z - \tau)\, \tilde{f}(\varphi, \tau) \qquad (39)$$

and expressing the source function of equation (37) $g = (1/cr) j_r$ as a Fourier series

$$g = \sum_m e^{im\varphi}\, g_m(\rho, z, \tau),$$

where

$$g_m = \frac{1}{2\pi c\rho\, \sqrt{\rho^2 + z^2}}\, \delta(\rho - \varepsilon\tau)\, \delta(z - \tau)\, \tilde{f}_m(\tau), \qquad (40)$$

one can immediately obtain the wave function in the form

$$\psi = \sum_m e^{im\varphi}\, \psi_m(\rho, z, \tau),$$

$$\psi_m = \frac{1}{4\pi^2 c\rho\sqrt{1+\varepsilon^2}} \int\limits_{\Phi_l}^{\Phi_{up}} d\rho' \frac{1}{\rho'^2} \times \tilde{f}_m\left(\frac{\rho'}{\varepsilon}\right) \frac{\cos m\theta(\varepsilon, \rho')}{\sin\theta(\varepsilon, \rho')}. \qquad (41)$$

Here the functions $\sin\theta(\varepsilon, \rho')$ and $\cos\theta(\varepsilon, \rho')$ and the integration limits are found in subsection 3.2. In the case $\varepsilon = 1$ the series coefficients are

$$\psi_m = \frac{1}{4\sqrt{2}\,\pi^2 c\rho} \int\limits_{\Phi_l}^{\Phi_{up}} d\tau' \frac{1}{\tau'^2}\, \tilde{f}_m(\tau')\, \frac{\cos m\theta(\tau')}{\sin\theta(\tau')}, \qquad (42)$$

where the integration limits are given in subsection 3.1, $\sin\theta(\tau')$ is defined by expression (24), and

$$\cos\theta = \frac{1}{2\rho\tau'} \left[\rho^2 + z^2 - \tau^2 + \tau'^2 + 2(\tau - z)\tau'\right].$$

Rewriting these expressions in the spherical coordinates and using expressions (36) and (37), we obtain the components of the magnetic field strength vector

$$H_\vartheta = i\, \frac{c}{\sin\vartheta} \sum_m m e^{im\varphi}\, \psi_m(r, \vartheta, \tau), \quad H_\varphi = -c\partial_\vartheta \sum_m e^{im\varphi}\, \psi_m(r, \vartheta, \tau). \qquad (43)$$

These expressions differ from the traditional expansions of the electromagnetic waves in terms of the spherical harmonics [8]. Kindred expansions were given for a circular source on the sphere expanding with the velocity of light in [2]. Calculation of the electric field components involves integration with respect to the time variable of expression (37) which is not, in general, a trivial problem.

Realization of the radial current source distributed on a circle belonging to a fictitious sphere expanding with a velocity greater than the velocity of light was discussed earlier (see [9,10], simplified models). The space-time structure of the electromagnetic field that is formed by the proposed sources substantially differs from the fields of traditional radiators. The physical conditions leading to generation of a current source on the expanding disk moving along a straight line with the velocity of light are not fully understood yet and need a further study.

5. Remarks

We have constructed a family of solutions of the wave equation defined by the fixed parameter $\beta = 1$. Conditions $\beta < 1$ and $\beta > 1$ define two other families of solutions with a more complicated space-time structure. In particular, the integration limits are found as coordinates of intersection points on the plane diagrams of hyperbolas (in the case $\beta < 1$) or ellipses (in the case $\beta > 1$) with straight lines. This circumstance substantially complicates construction of solutions.

Manuscript received June 27, 2006

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