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On the Description of Electromagnetic Signal Propagation through Conducting Media

Contents

Abstract

Dissipation of electromagnetic energy in conducting media takes place not only because of its absorption due to acceleration of charged particles by the electric field and their inelastic collisions with atoms, but also due to interaction of the wave's magnetic field with magnetic and electric dipole currents induced by those collisions in the atoms. The simplest way to describe this phenomenon is to use Ohm's law for magnetic dipole current as H. Harmuth suggested in his Modified Maxwell's equations.

1. Introduction

If you take a look at textbooks on classical electrodynamics, especially, the old ones, you may easily find that Maxwell's equations normally are formulated in such a way that they unify basic laws of electromagnetism discovered up to the date of the equations formulation (1860–65) and their publication (1873). Normally, so called constitutive equations are to be considered as an entire part of the set of electrodynamics equations to describe all electromagnetic (EM) phenomena in media. In particular, Ohm's law was always considered as one of these constitutive equations for conducting media. Ohm's law is present in the equations originally derived by Maxwell. However, at that period there very little was known about the microscopic structure of media. That is why these equations may be reasonably corrected in case of their failure in the adequate description of the phenomena that requires advanced knowledge of media microscopic structure.

Publication of the three papers by H. Harmuth [1–3] devoted to his modification of Maxwell's equations and the necessity to revise the notion of group velocity when studying electromagnetic signal propagation through conducting media has caused more than 10 years contradictory discussion in the IEEE Transactions on Electromagnetic Compatibility (see for example [4–9]). Those who treated themselves as well educated experts in electromagnetic theory did not accept the necessity of Harmuth's suggestion to modify Maxwell's equations for conducting media, and this was the reason for the discussion. Actually the motivation to modify the well accepted Maxwell's equations was their failure in describing EM signals propagation through a conducting medium in terms of non-divergent solutions within the formalism of initial-boundary-value problems with step-like initial conditions for EM field. It turned out that the associated magnetic field (but not the electric one!) has a divergent term when electric excitation is applied. In other words, the classical problem on propagation of the front of electromagnetic step-like (or pulse) waveform does not have a non-divergent solution if one is interested in obtaining a complete solution that supposes deriving and evaluating both electric and magnetic fields when studying such wave propagation through a conducting medium. When the magnetic analogue of the electric conducting current term has been inserted into the Faraday's law equation the above divergence disappeared even if one will put to zero the related magnetic conductivity coefficient in the obtained solution of the modified Maxwell's equations [1,2]. This has been shown by H. Harmuth and approved by other researchers [8,9] in terms of mathematics. Later on H. Harmuth noticed that even though magnetic charges have not been discovered yet there are many physical problems where one may consider dipole magnetic currents rather than monopole ones to make Maxwell's equations symmetric. In particular, this model has been applied to describe interstellar propagation of electromagnetic signals [10] and propagation of signals in non-conducting media with electric and magnetic dipole currents [11].

This short paper is devoted to another physical justification of Harmuth's modification of Maxwell's equations applicable to adequate description of electromagnetic properties of conducting media. I have noticed that when considering propagation of electromagnetic signals through conducting media where Ohm's law is applicable one must take into account the interaction of the magnetic field with dipole currents that are to be present in this case due to the neutral atoms deformations happened as a result of electrons or ions colliding with them. In this paper somewhat more detailed description of this idea is given.

2. A Simple Model of Media with Conducting (Monopole) and Dipole Currents, and the Modified Maxwell's Equations

According to all textbooks on electrodynamics two of Maxwell's equations, Ampere's law modified by Maxwell and Faraday's law of electromagnetic induction, are always valid for a medium containing no electric and magnetic charges either free or bound ones (the physical vacuum). At the same time, considering electromagnetic fields in a medium with electric charges one has to add so-called constitutive equations which establish relations between the fields and parameters of the medium. In this way, one may consider many modifications of the overall set of the equations governing electromagnetic fields in media. This is the commonly accepted and widely used standpoint in contemporary electrodynamics. A more general approach within the frame of classical physics consists in the combination of microscopic Maxwell-Lorentz equations with kinetic theory of neutral or partially ionized media [12].

There is an important problem of electromagnetic signal propagation through lossy media. If one needs to consider signal propagation through a lossy medium, such as gas, seawater, or dielectric solids, with dissipation of the *electromagnetic* field energy, one will necessarily need to describe this dissipation, and the simplest way to do that is to use Ohm's law connecting linearly electric field and electric current density associated with the motion of free electrons or ions. This may be done because of the existence of electrons as the carriers of electric monopole currents. At the same time, nobody introduces a similar law to describe losses associated with the magnetic field because the existence of free magnetic charges has not been proven. At this point it is worth to note that the electromagnetic field losses due to interaction with bounded charges or dipoles in dielectric and magnetic materials are usually described in terms of imaginary parts of dielectric permittivity and permeability associated with the electric and magnetic dipoles of the medium, respectively. Considering the propagation of electromagnetic signals in the form of rectangular pulses or step-functions through a lossy medium, H. Harmuth faced the problem of singularity in the expression for the magnetic field if the pulse was exited by electric excitation [1,2]. In order to go around this problem he suggested to *modify* Maxwell's equations via introduction into the Faraday's law a term proportionate to the magnetic field, i.e. he suggested to introduce Ohm's law for magnetic monopole or dipole current densities. Surprisingly, this step eliminated the singularity mentioned above even if the medium's magnetic conductivity will be put to zero in the solution obtained. This allowed him to investigate the propagation of signals through media with heavy losses. H. Harmuth called the equations obtained in this way *Modified Maxwell's equations*. Initially it was just a mathematical need which, by the way, changed the symmetry class of the equations under consideration (from $U(1)$ to $SU(2)$ as noticed by Terrence Barrett) but later on a magnetic dipole current, a completely physical quantity, was used to justify this mathematical modification.

In my opinion, there is one more physical justification for the need of that modification which I would like to explain briefly here though it deserves more detailed investigation. Let's recall the microscopic picture of the electric Ohm's law. The current density in a conducting medium is proportionate to the electric field strength because the electrons being accelerated by the electric field experience multiple inelastic collisions with heavy atoms[1], transferring portion of their kinetic energy to them and thereby heating the crystal lattice or separated atoms. However, the process of heating is the consequence of a huge number of microscopic (individual) processes of transforming the energy and *shape* of non-polarized neutral atoms or ions due to those collisions which leads to *varying of their dipole momentum in time*. This implies the generation of additional microscopic currents in the lossy medium, and the magnetic component of the field will necessarily interact with these currents (eventually transferring portion of its energy to them) which may be interpreted and described as Ohm's law for a magnetic current density. In this way, the conventional Ohm's law in a lossy medium is to be always supplemented with a magnetic Ohm's law! How strong will be this additional current is another reasonable problem. Normally, the related losses should be much less compared to the losses associated with the electric Ohm's law and therefore in many cases they may be ignored. However, this is not the case when studying the propagation of step-like (or rectangular pulse) signals through lossy media, in particular over extremely long distances since small effects will be accumulated during the long distance of propagation, and sooner or later they will make an appreciable contribution to the solution [10]. For the above reasons, Maxwell's equations with an added term for magnetic current density should be considered as one of the possible modifications of Maxwell's equations, but because of the exceptional importance of the particular case of lossy media the term *Modified Maxwell's equations* may be applied and reasonably used.

3. Propagation of Signals through Conducting Media with Dipole Currents

Consider propagation of EM wave through conducting media which also admit the existence of dipole currents both electric and magnetic ones. The medium is considered within the frame of a *classical microscopic* model using the description of an atom as a combination of electric and magnetic dipoles mechanically bounded. Those dipoles produce electric and magnetic dipole currents under the electromagnetic field action that is to be calculated in a self-consistent way. Besides, we will take into account a dipole current induced due to collisions of accelerated charged particles with atoms of the medium. We describe the model under consideration, formulation of initial-boundary value problem for the modified equation and the method for its solution in the time domain.

The propagation of EM waves is governed by Maxwell's equations. However, those equations should be modified according to the properties of the media in which EM waves propagate. We admit the presence of both the conductivity currents and dipole ones in the medium under consideration. This may occur in metals, in partially ionized gases, in semiconductors with current injection, etc. The electric field strength will pull the positive proton and the negative electron slightly apart and produce an electric dipole. An electric dipole current flows while this pulling apart is in progress and also when the electric field strength drops to zero and the atom returns to its original, non-polarized state. Similarly, an atom may have a magnetic momentum like a little bar magnet. A magnetic field strength will rotate the atoms to make them line up with the field strength. Magnetic dipole current flows while this rotation is in progress and also when the magnetic field strength drops to zero and the magnetic dipoles return to their original random orientation. Similar behavior may happen with atom dipoles due to collisions with a charged particle. Actually in this case in stead of the electric field of the wave the Coulomb field of the charged particle deforms the above dipoles. So, we have to take into account the reaction of the medium onto the action of the EM field of the propagating signal and the collisions of charged particles with neutral atoms. Conventional approaches to solutions of that problem suppose that both the intrinsic characteristic time of the media and its relaxation time are much smaller the characteristic time of the EM field variation. Besides, they also suppose performing space averaging of the EM fields introducing into consideration both electric and magnetic flux densities. Since in our case we have very fast variation of the EM signal fields those methods are not applicable anymore. In order to solve the problem of signal propagation through conducting media one has to modify Maxwell equations using both electric and magnetic dipole current densities [10]. This implies description of the medium in the frame of a *microscopic* approach using the representation of an atom as a combination of electric and magnetic dipoles. Those dipoles produce *electric and magnetic dipole currents* due to the EM field and the above collisions which are to be calculated in a self-consistent way. Self-consistent systems containing both the modified Maxwell equations for conducting media and equations for the dipole current densities evolution under the EM field and collisions can be written in the following form:

[1] Actually there are three main scattering mechanisms in conducting solids: electron scattering by impurity atoms or other inhomogeneities of the lattice; scattering by lattice phonons and electron-electron scattering.

$$-\mathrm{rot}\,\vec{E} = \mu\frac{\partial\vec{H}}{\partial t} + \langle\vec{g}_m^c\rangle + \vec{g}_m, \tag{1}$$

$$\mathrm{rot}\,\vec{H} = \varepsilon\frac{\partial\vec{E}}{\partial t} + \sigma\vec{E} + \langle\vec{g}_e^c\rangle + \vec{g}_e, \tag{2}$$

$$\varepsilon\,\mathrm{div}\,E = 0, \qquad \mu\mathrm{div}\,\vec{H} = 0, \tag{3}$$

$$\vec{g}_e + \tau_{mp}\frac{\partial\vec{g}_e}{\partial t} + \frac{\tau_{mp}}{\tau^2}\int\vec{g}_e dt = \sigma_p\vec{E}, \tag{4}$$

$$\vec{g}_m + \tau_{mp}\frac{\partial\vec{g}_m}{\partial t} + \frac{\tau_{mp}}{\tau^2}\int\vec{g}_e dt = 2s_p\vec{H}, \tag{5}$$

$$\langle\vec{g}_e^c\rangle = e\,n\int\mathbf{v}_e^c f_{ab}(r,p,R,t)drdp, \tag{6}$$

$$\langle\vec{g}_m^c\rangle = e\,n\int\mathbf{v}_m^c f_{ab}(r,p,R,t)drdp, \tag{7}$$

where $\vec{E}$ and $\vec{H}$ are electric and magnetic fields; $\sigma_p = N_0e^2\tau_{mp}/m$ and $s_p = N_0q_m^2\tau_{mp}/m$ are electric and magnetic dipole current conductances; g_e and g_m are electric and magnetic dipole current densities induced by the EM field; $< \vec{g}_e^c >$ and $< \vec{g}_m^c >$ are statistically averaged electric and magnetic dipole current densities induced by the collisions; ε and μ are permittivity and permeability of the vacuum; e and m are charge and mass of the electron; m_{m0} and $q_m = \dfrac{\mu m_{m0}}{2r}$ are atom magnetic dipole moment and fictitious magnetic charge; τ_{mp} and τ are the relaxation time and period of resonant frequency of the dipole-oscillator used as the model for atom of the medium.

The probability distribution function $f_{ab}(r,p,R,t)$ in Eq.(6) and Eq.(7) depends on local coordinate r and impulse p within an atom at its position R and reflects the distribution of atom dipoles disturbed due to collisions of accelerated charged particles of the conductivity current with neutral atoms compound of charged particles a and b. This function is to be found from the kinetic equation for system of charged particles, atoms and electromagnetic fields with integral of collisions that takes into account the probability of the above collisions and the relaxation of the disturbed dipole currents according to Eq. (4) and Eq. (5). The currents (6) and (7) depend on both electric and magnetic fields and for a weak field (when Ohm's law is valid) the dependences should be linear. If the relaxation time for the atom dipoles due to collisions is much smaller of that due to electromagnetic field influence we may rewrite Eq. (1) and Eq. (2) as follows:

$$\mathrm{rot}\,\vec{E} = \mu\frac{\partial\vec{H}}{\partial t} + s_p^c\vec{H} + \vec{g}_m, \tag{8}$$

$$\mathrm{rot}\,\vec{H} = \varepsilon\frac{\partial\vec{E}}{\partial t} + (\sigma + \sigma_p^c)\vec{E} + \vec{g}_e, \tag{9}$$

where σ_p^c and s_p^c are electric and magnetic dipole collision conductivities, respectively.

In this way, in addition to the conductivity current and dipole currents in conducting media one has takes into account the interaction of electric and magnetic fields with dipole currents caused by collisions of charged particles with neutral atoms of conducting media. This will result in an additional dissipation of electromagnetic energy. Usually contributions by dipole currents are smaller than by conductivity currents. That is why we may neglect "collision dipole current" in Eq. (9), but it should be taken into account in Eq. (8). This means that one has to take into account the variation of the atoms electric and/or magnetic dipoles due to inelastic collisions with charged particles that will cause an electric dipole current of atoms to flow. The latter will be a reason for EM energy dissipation not only due to accelerating of electric charges, but also due to the wave's magnetic field interaction with the above electric dipole current. This means that whenever we have to consider propagation of electromagnetic waves through conducting media we have to take into account the fact that dissipation of electromagnetic energy takes place not only because of its absorption due to the interaction of charged particles with the wave's electric field and their further inelastic collisions with atoms, but also due to the interaction of the wave's magnetic fields with magnetic and electric dipole currents induced by those collisions in atoms. The simplest way to do that is to use Ohm's law for magnetic dipole current as H.Harmuth suggested in his seminal papers [1,2] and his following books.

Let consider as an example a planar, transverse electromagnetic (TEM) wave propagation through the above conducting medium along the direction y. Solution of that problem may be readily derived if one follows the procedure described in [10]. A TEM planar wave requires

$$E_x = H_y = 0, \quad E_x = E_z = E, \quad H_x = -H_z = H.$$

With this simplification, the above system of partial integro-differential equations is reduced to the sixth order linear PDE for either electric E or magnetic H field components [10]. In order to investigate the propagation of EM signal through the above medium one has to solve initial-boundary-value problem for that equation with the following initial ($t = 0$) condition

$$E(y,0) = \frac{\partial^n E(y,0)}{\partial^n} = \int\frac{\partial^2 E}{\partial t^2}dt = \int\int\frac{\partial^2 E}{\partial t^2}dtdt' \tag{10}$$

for $n = 1,2,3$ and the boundary ($y = 0$) conditions

$$E(\infty,t) < \infty, \quad E(0,t) = E_0 S(t)(1 - e^{-t/\tau_s}),$$

where $S(t)$ is Heaviside's step function:

$$S(t) = \begin{cases} 0, & \text{for} \quad t < 0, \\ 1, & \text{for} \quad t \geq 0. \end{cases}.$$

The above problem is solved [10] by the variables separation method. The solution has the following form:

$$E(y,1) = E_0 \left[w(y,t) + e^{-(y/L + t/\tau_s)} \right], \qquad (11)$$

where

$$w(y,t) = \int\limits_0^{\infty} \left(\sum_{i=1}^{6} A_i(k) e^{\gamma_i(k)t/\tau_{mp}} \right) \sin(2\pi k y) dk$$

and k is the separation constant; $\gamma_i(k)$ are six roots of the characteristic equation for PDE derived from Eqs. (4),(5),(8),(9); $A_i(k)$ are six constants to be determined by six initial conditions (10). The expressions for $L(\tau_{mp}, \tau)$, $\gamma_i(k)$ and $A_i(k)$ have an explicit, but rather complicated form and can be found in the monograph [10] with the minor corrections for collision dipoles and conductivity currents. Here we only note that because of the higher symmetry of the modified Maxwell equations (8),(9) we are able to find analytical solutions for the sixth order characteristic equation. This enabled us to both performing the analysis of the Eq. (11) integrand as function of k and developing efficient algorithms for numerical evaluation of the electric and magnetic fields of propagating signals.

4. Conclusion

We have shown that whenever one has to consider the propagation of electromagnetic waves through conducting media one has to take into account the fact that dissipation of electromagnetic energy takes place not only because of its absorption due to interaction of charged particles with the wave's electric field and their further inelastic collisions with atoms, but also due to interaction of the wave's magnetic fields with magnetic and electric dipole currents induced by those collisions in atoms. The simplest way to do that was to use Ohm's law for magnetic dipole current as H. Harmuth suggested in his seminal papers [1,2] and his following books [10,15–18].

Concluding my paper I would like to come back to the story of publications of Harmuth's papers [1,2] and some consequences of that. As I mentioned above, these papers have initiated more than 10 years discussion whether it is necessary or useless to modify Maxwell's equations for conducting media: some authors supported H.Harmuth's ideas, while others noted that the idea is not a new one since many other authors used fictitious magnetic currents in Maxwell's equations to introduce symmetry and make solutions finding much easier. Finally, there were those who were very much against this modification of Maxwell's equations, considering it not just useless, but also unacceptable. They opposed that idea as strongly as did those "leading experts" mentioned in the very first page of this Issue. I think just those experts were insisting on punishing the person who allowed the publication of those papers: Richard B. Schultz the Editor of IEEE Transaction on Electromagnetic Compatibility. He retired as Editor of the above journal shortly after those publications. This is not the only such case in the history of physics and may be not only of physics, but also other fields of human mental activity. However, it is interesting to recall briefly that a similar story happened to Oliver Heaviside 100 years before Harmuth's publications. Nowadays we respect Oliver Heaviside for his amazing and remarkable contributions to electromagnetism, and consider him as a well-recognized scientist and "telegraphist" [19]. He was well ahead of his time in many issues of electrodynamics and signal propagation in telegraph cables. He was the first who formulated Maxwell's equations in the form we use it now; he created the theory of signal propagation in cables; he predicted Cherenkov's radiation and much more. At the end of the 19th century, telephone and telegraph communication was a big business which also stimulated the related research focused on the increase of communications distance and the enhancement of its quality. All experts of that time widely believed that signals transmitted through a cable were nothing else but electric currents flowing *in* its conducting wire similar to a liquid in a pipe. O. Heaviside was the first who figured out, that an electromagnetic signal in a coaxial cable is not a current flowing in a conducting wire, but an electromagnetic field concentrated between the wire and the shield. This helped him to understand how to enhance distortionless propagation of EM signals in cables and to increase the distance of the telephone communications. However, in 1887 when he "...first suggested adding extra inductance to telephone lines the idea struck most engineers as absurd" [19,p. 137]. Heaviside's attempt to publish his idea in the *Electrician* Journal faced the problem of getting clearance for publication from his boss W.H. Preece, chief of the Post Office telegraph engineers, who stopped this paper from publication since he disagreed with Heaviside's idea. As a leading expert in W. Thomson's theory of electric signal propagation in the conducting wire (which was wrong as we know now) he considered Heaviside's idea as completely wrong and absurd. However Heavisde managed to publish his paper by parts with the support of Mr. Biggs, the editor of *Electrician* who published Heaviside's articles "in spite of most strenuous opposition by proprietors and every member of the staff" [19,p. 142].

It happened in the period between April and September of 1887, and "early in October 1887 he was abruptly removed as Editor of the Electrician"

[19,p. 142]. A familiar story, is it not?

It is also interesting to recall that "...when loading coils were finally introduced commercially around 1900 in America, their success helped make a number of scientists, engineers, and corporations, — conspicuously not including Heaviside — very rich. There is thus a special irony in the fact that when Heaviside first proposed inductive loading in 1887, British telephone engineers rejected his suggestion outright" [19,p. 137].

I would recommend to read Harmuth's books rather carefully with keeping in mind the above story.

To those who are interested in the details of interesting and not easy work of the "Maxwellians" in the 19th century I would strongly recommend to read the book [19].

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