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Induction MHD Flow near Cylinder

Contents

1. Introduction **4**
1.1. Forms of stability loss of falling sheet 4
1.2. Sheet circulation paradox 6

2. Problem formulation **8**

3. Function of current **8**

4. Turbulent force action **9**

5. Euler-d'Alambert paradox **10**

6. Hydrodynamic moment turbulent part **11**

7. Integral representations **12**

8. Effect of inertia **12**

9. Ellipse. Preliminary formulae **14**

10. Impact of force on ellipse **15**

11. Effect of moment **16**

12. Particular cases **17**

13. Moment-free flow-around **17**

14. Effect of moment on plate **18**

15. Reduction of formulae **19**

16. Conclusions **20**

Abstract

The derivation is made of integral representations of the turbulent and inertial parts of the hydroelectromagnetic force and moment that act on contour in the flow. In the absence of EMF the turbulent hydrodynamic force determines action on the plane contour made by the potential circulation-free flow of the ideal fluid, thereby disposing of the Euler-d'Alambert paradox in the ideal fluid theory. The effects of force and moment on the ellipse are determined for the turbulent EMF of non-viscous fluid at an arbitrary value of the angle of incidence and at the angle between velocity of the running flow at infinity and vector of the applied magnetic field induction. The conditions are found such at which the moment collapsing the ellipse during non-symmetrical flow-around is zero due to a factor which is other than being of the hydrodynamicnature.

1. Introduction

The flow of electricity-conductive fluid in magnetic field is the subject matter of research by our compatriot [1] and foreign [7]. scientists. A great number of those studies are aimed at research on the hydroelectromagnetic effects produced on contour placed in the current-conductive fluid under an applied magnetic field, in particular, at definition of the hydrodynamic action of the ideal fluid on the contour in the absence of EMF. Such problems have no solution for smooth contours (the Euler-d'Alambert paradox [4]) If the contour has an angular point, then the solution of the above problems involves the use of circulation currents [4, 5, etc.]. The Authors of this paper arrived at a conclusion, relying on the example of studies made on forms of the stability loss of falling sheet (see below), that the use of the circulation currents does not allow for description to be made of one of the most commonly employed forms of the loss of stability of falling plate. While using the results of the reference [3], this result is easy to generalize to such a case where the Zhukovsky profile is collapsed in the fluid. The results of the present work in its particular cases dispose of the Euler-d'Alambert paradox and preclude the necessity of using the circulation while defining the effects on contours in the ideal fluid flow.

The original equations in this work are as follows:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\boldsymbol{\nabla})\mathbf{v} = -\frac{1}{\rho}\boldsymbol{\nabla}p - \kappa(\mathbf{v}-\mathbf{V}) + \mathbf{F}, \qquad \boldsymbol{\nabla}\cdot\mathbf{v} = 0 \tag{1}$$

characterizing the main [5] turbulent flow of the ideal fluid. In the equations (1) $\mathbf{V}$ – the fluid velocity at infinity (or velocity of the capacity with fluid), κ – the phenomenological parameter characterizing the turbulent resistance of the fluid [2, 3].

1.1. Forms of stability loss of falling sheet

We shall have to use two very simple formulae derived below to describe the hydrodynamic effect of a non-conductive ideal fluid flow on plate. We shall use the formulae (66) and (81) to address the problem which is new to the hydrodynamics: description of the behavior of a sheet falling down through calm air in its path.

If a rectangular thin cardboard sheet is positioned horizontally and released at the initial moment without a starting velocity, it moves for a certain time in the vertical while keeping its horizontal position and, then, one of the axes of the rectangle begins to oscillate near the vertical remaining perpendicular to it (i.e. the sheet axis remains being horizontal). The plane of the sheet also begins to perform small oscillations near the horizontal axis of the rectangle. This form of the loss of stability of falling sheet is termed as *oscillatory*.

If the same sheet is positioned vertically at the initial moment and comes down, too, without a starting velocity, it glides for a certain time in the vertical remaining in the vertical position, and then it begins to rotate round the larger of its axes. The larger axis deviates from the original vertical and falls down while remaining being horizontal, the horizontal deviation of the rotation axis off the initial vertical being considerably larger than its vertical displacement. This form of the loss of stability of falling sheet is termed as *rotational*. Both of these forms of the loss of stability of falling sheet are known well from observation of leaves shed by trees.

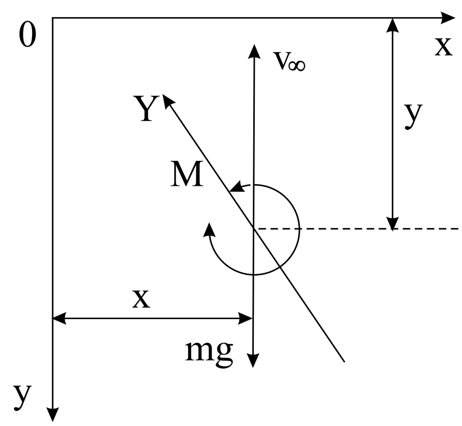

Fig. 1. Forces acting on sheet.

The falling sheet is simulated as a uniform infinite plate of the width $2a$. Its unit length has the mass m and the moment of inertia J relative to the centre-of-gravity. The position of the sheet in the rectangular axes Oxy that has the horizontal axis Ox and the axis Oy, which is directed against the gravity $m\mathbf{g}$, is determined by the coordinates x, y of its centre -of-gravity and the angle of turn φ of the normal to the plate from the vertical Oy (Fig. 1).

It is demonstrated above that, besides the gravity $m\mathbf{g}$ the sheet is subjected from the direction of the air to the force Y, which is normal to the sheet plane at any of its orientation relative to the direction of the gravity, and the moment M relative to the centre-of-gravity of the sheet, as determined by the formulae (66) (81) $S = 0$, which have the following form in the accepted herewith designations:

$$Y = \pi\rho\kappa a^2 v\cos\varphi, \qquad v = \sqrt{\dot{x}^2 + \dot{y}^2}, \qquad M = -\pi\rho a^2 v^2 \cos\varphi\sin\varphi \tag{2}$$

ρ – the air density, κ – the coefficient of turbulent resistance in the sheet-excited aerial flow.

Neglecting at once the associated mass, we write down the equations of the sheet motion (Fig. 1):

$$\begin{aligned} m\ddot{x} &= -\pi\rho\kappa a^2\sqrt{\dot{x}^2+\dot{y}^2}\cos\varphi\sin\varphi, \\ m\ddot{y} &= mg-\pi\rho\kappa a^2\sqrt{\dot{x}^2+\dot{y}^2}\cos^2\varphi, \\ J\ddot{\varphi} &= -\pi\rho a^2(\dot{x}^2+\dot{y}^2)\cos\varphi\sin\varphi. \end{aligned} \tag{3}$$

The set of equations (3) has the particular solution:

$$\varphi=0, \quad x=0, \quad y=v_0t, \quad v_0=\frac{mg}{\pi\rho\kappa a^2} \tag{4}$$

which signifies that the sheet can keep falling uniformly and rectilinearly along the force of gravity down from its initial horizontal position at the constant velocity v_0, , remaining in the horizontal attitude. Just how balanced is this fall?

Let us linearize the equations (3) to the solution (4):

$$\begin{aligned} &\ddot{x}+g\varphi=0, \quad v_0\ddot{y}+g\dot{y}=0, \\ &\ddot{\varphi}+\omega^2\varphi=0, \quad \omega=\frac{mg}{\kappa a\sqrt{\pi\rho J}} \end{aligned}$$

We shall derive herewith the parametric equations of the falling sheet path:

$$\begin{aligned} &x=C_3+C_4t+\frac{g}{\omega^2}\left(C_1\cos\omega t+C_2\sin\omega t\right), \\ &\varphi=C_1\cos\omega t+C_2\sin\omega t y=C_5+C_6\exp\left(-\frac{gt}{v_0}\right) \\ &\quad (C_k=const). \end{aligned}$$

At $C_4\neq 0$ the sheet centre-of-gravity path can well deviate no matter how far from the initial vertical $x(t)$ of the sheet oscillates near the slanting ($C_4\neq 0$) or vertical ($C_4=0$) ray with the constant amplitude and frequency ø. At the same frequency and amplitude the normal to the sheet plane ($\varphi(t)$) oscillates near the vertical. A narrow sheet with a smaller moment of inertia oscillates with a lesser period than a broad one. In a moist, denser air the period of the sheet oscillations is greater than in dry air.

This kind of sheet-falling in calm air is known well from practical observations. The oscillatory fall is observable by everyone who will carefully release, for example, a library record card or a small calendar from the horizontal position. The result as produced physically stands to signify that (due to (4)) the oscillatory nature of the motion of the sheet centre-of-gravity which is normal toward its plane corresponds more to the uniform, rather than equi-accelerated, fall of the sheet.

In due course, the path of the falling sheet will asymptotically assume the position in which the centre-of-gravity of the sheet oscillates in the horizontal plane $y=C_5=const$ and falls down no more. This position proves unstable again which is supported by observations of the rectangular cardboard sheet released from the initial horizontal attitude. Let us leave the description of this subsequent phase of sheet-falling to more enduring researchers.

Besides the solution (4) the set (3) admits the following solution

$$\varphi=\frac{\pi}{2}, \quad x=0, \quad y=\frac{1}{2}gt^2 \tag{5}$$

which means that the sheet can fall in acceleration along the vertical while retaining its plane vertical. By assuming that

$$\varphi=\psi+\frac{\pi}{2}, \quad y=\eta+\frac{1}{2}gt^2$$

let us linearize the equations (3) to the solution (5). Then the sheet motion equations are

$$m\ddot{x}=\pi\rho\kappa a^2gt\psi, \quad J\ddot{\psi}=\pi\rho a^2g^2t^2\psi, \quad \ddot{\eta}=0.$$

Hence at the condition $x(0)=0$, $\eta(0)=0$, $\psi(0)=0$ we shall find that:

$$\begin{aligned} &\psi=C_1\sqrt{t}I_{1/4}\left(\frac{1}{2}agt^2\sqrt{\frac{\pi\rho}{J}}\right) \\ &x=C_2t+\frac{\pi\rho\kappa a^2g}{m}\int_0^t\int_0^\tau z\psi(z)dzd\tau \\ &\eta=C_3t, \end{aligned} \tag{6}$$

$I_\nu(z)$ – the McDonald function that grows exponentially at infinity.

In this way, the angle ψ increases monotonously (exponentially) over time, i.e. the sheet gets accelerated rotation around its own axis. The rotation axis itself deviates from its initial position in the horizontal direction x *following the exponential law* and moves downward following the power law (5), i.e. considerably *slower*. A narrow sheet with a lesser moment of inertia ($J=const\cdot a^2$) rotates faster than a broad one. A light sheet with a smaller mass deviates farther off than a heavier one.

This portion of the qualitative picture of the rotational form of the loss of stability of falling sheet agrees completely with observations. The only thing that is not true is just the dependence (6) of the turning angle of the plane of the plate ψ on the air density ρ while it's getting denser the angular velocity of the falling sheet rotation $\dot{\psi}$ must not increase over time. However, the nature of this inaccuracy is not associated with the Euler turbulent resistance equations [3] and their consequences, i.e. the formulae (2), on which the research on motion of the sheet is based. The mistake is due to the unaccounted associated moment of the sheet inertia. (While determining the air response to the sheet it should be taken into account that it might rotate with acceleration. This kind of accountability transforms the argument of the function $I_{1/4}$ in the formula (6)into a fractionally linear function ρ so that there is no unlimited growth in the McDonald function with increasing density ρ).

1.2. Sheet circulation paradox

Let us now resolve the problem of falling sheet on the base of the classical expressions for the force acting on the sheet from air as determined during circulation flow passing round plate. In this case, the air acts on the sheet with such force and moment that are relative to the centre-of-gravity with projections [4]:

$$\begin{aligned} X &= 2\pi\rho a v^2 \cos^2\varphi, \\ Y &= 2\pi\rho a v^2 \sin\varphi\cos\varphi \\ M &= -\pi\rho a^2 v^2 \sin\varphi\cos\varphi, \\ v^2 &= \dot{x}^2 + \dot{y}^2 \end{aligned}$$

the directions of which are shown in Fig. 1. This is to say that the sheet motion equations

$$\begin{aligned} m\ddot{x} &= -X\cos\varphi - Y\sin\varphi, \\ m\ddot{y} &= X\sin\varphi - Y\cos\varphi + mg, \\ J\ddot{\varphi} &= M \end{aligned}$$

in this case have the following form (the associated mass is neglected)

$$\begin{aligned} m\ddot{x} &= -2\pi\rho a(\dot{x}^2 + \dot{y}^2)\cos\varphi, \\ \ddot{y} &= g \\ J\ddot{\varphi} &= -\pi\rho a^2(\dot{x}^2 + \dot{y}^2)\sin\varphi\cos\varphi. \end{aligned} \tag{7}$$

The set of equations (7) has no solution of the kind $\varphi = 0$, $x = 0$, $y = y(t)$, which would determine the initial falling sheet path portion with such horizontal position of its plane that would have been prior to appearance of the oscillatory form of the loss of stability. This is obvious.

Nor does the set of equations (7) have the continuous particular solution of the kind $\varphi = 0$, $x = x(t)$, $y = y(t)$.

Proof. Let us assume that in the formula (7) $\varphi = 0$, $\dot{x} = z(t)$, we take into account that the velocity $\dot{y} = gt + y_0$, and designate

$$\frac{2\pi\rho a}{m} = A^2, \quad \frac{2\pi\rho a}{m} g^2 = B^2$$

Then the function $z(t)$ satisfies the equation

$$\dot{z} + A^2 z^2 + B^2 t^2 = 0$$

Let

$$z = \frac{\dot{u}}{A^2 u}.$$

Then in order to determine the function $u(t)$ we shall obtain a linear equation of the second order:

$$\ddot{u} + A^2 B^2 t^2 u = 0.$$

The general solution of the equation derived is as follows:

$$u = \sqrt{t}\left[C_1 J_{1/4}\left(\frac{1}{2}ABt^2\right) + C_2 J_{-1/4}\left(\frac{1}{2}ABt^2\right)\right]$$

J_ν – the Bessel function.

At any values of C_1 and C_2 the Bessel's function found in the brackets has the countable set of the positive roots that do not coincide with the roots of the function $\dot{u}(t)$. This is to mean that the velocity

$$\dot{x} = z = \frac{\dot{u}}{A^2 u}$$

is not the continuous time function, i.e. *the solution $\varphi = 0$ is devoid of physical meaning.* It is also clear that there are no continuity solutions for the set (7)that would be close to $\varphi = 0$. It follows that the small oscillations at the angle $\varphi(t)$ are not described by the set (7) Thus, the oscillatory form of the loss of stability of the horizontal falling of the sheet is not feasible from the point of view of the set (7) (i.e. from the point of view of the circulation continuous flow passing round plate).

On the other hand, the set (7) allows for a particular solution

$$x = 0, \quad y = \frac{1}{2}gt^2, \quad \varphi = \frac{\pi}{2} \tag{8}$$

which corresponds to the vertical release of the sheet from the initial vertical position $x = y = 0$, $\varphi = \pi/2$ without the starting velocity.

In order to study the form of the loss of stability of the motion (8), from the point of view of (7), we shall linearize this set to the solution (8), assuming that $\varphi = \psi + \pi/2$. Then to determine the functions $x(t)$ and $\psi(t)$ we shall obtain the set of equations

$$m\ddot{x} = 2\pi\rho a g^2 t^2 \psi, \quad J\ddot{\psi} = \pi\rho a^2 g^2 t^2 \psi.$$

From which

$$\begin{aligned} \psi &= C_1\sqrt{t} I_{1/4}\left(\frac{1}{2}agt^2\sqrt{\frac{\pi\rho}{J}}\right), \\ x &= C_2 t + \frac{2\pi\rho a g^2}{m}\int_0^t\int_0^\tau z^2\psi(z)\,dz\,d\tau. \end{aligned} \tag{9}$$

The initial conditions of the release of the sheet $x(0) = 0$, $\psi(0) = 0$ are now accounted for.

Qualitatively, the solution (9) is now close to the solution (6) which takes into account the expression (8) for $y(t)$. In this way, if one models the air, through which the sheet is falling as ideal fluid, and determines its effect on the sheet on the base of the appropriate formulae for the circulation continuous flow-around, then the oscillatory form of the loss of stability of the initial horizontal position of the falling sheet cannot be described by this model, whereas the rotational form of the loss of stability is applicable.

What is the underlying reason for this paradox? Why does one of the forms of the loss of stability of falling sheet (rotational), described by the circulation continuous model of impact on the body, have all the observable parameters agreeing, whereas the other form of the loss of stability (oscillatory) defies any description at all?

There is no use seeking the cause in the air viscosity, in the discontinuous nature of the flow or in the plane sheet model being too crude, since the description of the rotational form of the loss of stability brings out the same drawbacks, and, still, the quality of description by the circulation model is quite satisfactory.

The underlying cause of the paradox lies in the assumption of the feasibility of definition of the hydrodynamic force of the action of air on the sheet by using the notion of circulation.

It is exactly the expression of force based on the notion "circulation" that precludes the set (7) from describing the oscillatory form of the loss of stability. While for the rotational form of the loss of stability the basic equation in the set (8) is the one used to determine the angular acceleration. The derivation of the "rotational" equation does not involve the mechanism of circulation, the hydrodynamic moment in the circulation flow being the same as in the circulation-free one. The moment is the same both in the derivation of the moment equation (7),and in the derivation of the equations (3) with the air microstructure taken into account. For this reason, the formulae (9) and (6), which describe the rotational loss of stability, are very close, indeed.

The main conclusion ensuing from the considered paradox is such that the circulation currents should be left out from those parts of the ideal uncompressible fluid hydrodynamics that deal with issues of the definition of the hydrodynamic effects on 1D closed contours in the running fluid flow.

The research done within this section indicates that the Euler turbulent resistance equations model is ultimately simple to describe many of the phenomena of technical or natural origin for the description of which there are no even approaches in the classical hydromechanics. The oscillatory form of the loss of stability of the horizontal sheet fall is not possible from the point of view of the set (7) (that is to say, from the standpoint of circulation continuous flow passing round plates).

On the other hand, the set (7) admits a particularsolution:

$$x = 0, \quad y = \frac{1}{2}gt^2, \quad \varphi = \frac{\pi}{2} \tag{10}$$

which corresponds to the vertical release of the sheet from the initial vertical position $x = y = 0$, $\varphi = \pi/2$ without the starting velocity.

In order to study the form of the loss of stability of the motion (10), from the point of view of the set (7), we shall linearize this set to the solution (10), assuming that $\varphi = \psi + \pi/2$. Then, to determine the functions $x(t)$ and $\psi(t)$ we shall obtain the following set of equations:

$$m\ddot{x} = 2\pi\rho a g^2 t^2 \psi, \quad J\ddot{\psi} = \pi\rho a^2 g^2 t^2 \psi$$

from which

$$\begin{aligned} \psi &= C_1\sqrt{t}I_{1/4}\left(\frac{1}{2}agt^2\sqrt{\frac{\pi\rho}{J}}\right), \\ x &= C_2 t + \frac{2\pi\rho a g^2}{m}\int\limits_0^t\int\limits_0^\tau z^2\psi(z)dzd\tau. \end{aligned} \tag{11}$$

The initial conditions $x(0) = 0$, $\psi(0) = 0$ of the release of the sheet are now accounted for.

Qualitatively, the solution (11) is close to the solution (6) taking into account the expression (10) for $y(t)$.. In this way, if one models the air, through which the sheet is falling as ideal fluid, and determines its effect on the sheet on the base of the appropriate formulae for the circulation continuous flow-around, then the oscillatory form of the loss of stability of the initial horizontal position of the falling sheet cannot be described by this model, whereas the rotational form of the loss of stability is applicable.

What is the underlying reason for this paradox? Why does one of the forms of the loss of stability of falling sheet (rotational), described by the circulation continuous model of impact on the body, have all the observable parameters agreeing, whereas the other form of the loss of stability (oscillatory) defies any description at all?

There is no use seeking the cause in the air viscosity, in the discontinuous nature of the flow or in the plane sheet model being too crude, since the description of the rotational form of the loss of stability brings out the same drawbacks, and, still, the quality of description by the circulation model is quite satisfactory.

The underlying cause of the paradox lies in the assumption of the feasibility of definition of the hydrodynamic force of the action of air on the sheet by using the notion of circulation.

It is exactly the expression of force based on the notion "circulation" that precludes the set (7) from describing the oscillatory form of the loss of stability. While for the rotational form of the loss of stability the basic equation in the set (10) is the one to determine the angular acceleration. The derivation of the "rotational" equation does not involve the mechanism of circulation, the hydrodynamic moment in the circulation flow being the same as in the circulation-free one. The moment is the same both in the derivation of the moment equation (7), and in the derivation of the equations (3) with the air microstructure taken into account. For this very reason, the formulae (11) and (6), which describe the rotational loss of stability, are very close, indeed.

The main conclusion ensuing from the considered paradox is such that the circulation currents should be left out from those parts of the ideal uncompressible fluid hydrodynamics that deal with issues of the definition of hydrodynamic effects on 1D closed contours in the running fluid flow.

The research carried out to date indicates that the model of the Euler turbulent resistance equations is ultimately simple to describe many phenomena of technical or natural origin for the description of which there are no even approaches in the classical hydromechanics.

2. Problem formulation

We shall continue our studies [2] on the turbulent flow of conductive fluid in magnetic field. The flow runs in the plane Oxy, where O – the centre-of-gravity of the area Ω, that is limited dimensionally L_z by the contour L' round which the flow passes and which is length-scaled as L_* [2]. The electric field potential is $\psi = -Cz + C_1$, where $(\beta = -\alpha + \pi/2)$ [2]

$$C = \frac{1}{\lambda^2} \cdot \left.\frac{\partial\varphi}{\partial y}\right|_{x^2+y^2\to\infty} = \sin\alpha \qquad (12)$$

φ – the velocity potential, α – the angle between the velocity $\mathbf{V}^0$ of the flow at infinity and the induction $\mathbf{B}^0$ of the applied magnetic field along which the value Ox is directed. The boundary condition for φ:

$$\left.\frac{\partial\varphi}{\partial n}\right|_{L_\zeta} = -\lambda S \cos(n,\eta)\sin\alpha\Big|_{L_\zeta} \qquad (13)$$

where L_ζ – the map of the contour L_z at appropriate affine transformation, S and $\lambda = \sqrt{\kappa L_*^2/\nu}$ – the Stewart number [2]. The function ψ is independent of x and y, i.e. on the same contour $\partial\psi/\partial n = 0$ automatically. The function φ in the plane $\zeta = \xi + i\eta$ outside the contour L_ζ satisfies the Laplace equation

$$\nabla^2\varphi \equiv \frac{\partial^2\varphi}{\partial\xi^2} + \frac{\partial^2\varphi}{\partial\eta^2} = 0 \qquad (14)$$

the condition (13) on the contour L_ζ and the conditions at infinity [2] are:

$$\begin{aligned} \left.\frac{\partial\varphi}{\partial\xi}\right|_{\xi^2+\eta^2\to\infty} &= \lambda^2\sqrt{\lambda^2+S}\cos\alpha, \\ \left.\frac{\partial\varphi}{\partial\eta}\right|_{\xi^2+\eta^2\to\infty} &= \lambda^3\sin\alpha \end{aligned} \qquad (15)$$

The projections of velocity and density of the current are expressed in terms of φ:

$$\begin{aligned} v_x &= \frac{1}{\lambda^2}\frac{\partial\varphi}{\partial x}, \\ v_y &= \frac{1}{\lambda^2+S}\left(\frac{\partial\varphi}{\partial y} + S\sin\alpha\right), \\ v_z &= 0 \\ j_x &= j_y = 0, \\ j_z &= \frac{1}{\lambda^2+S}\left(\lambda^3\sin\alpha - \frac{\partial\varphi}{\partial y}\right). \end{aligned} \qquad (16)$$

Considering the flow passing round plane contours, the velocity field and electric current density are determinable only by the relations (13) – (16) regardless of whether the contour is conductor or dielectric, because the electric current potential is not included in the limiting conditions of the problem. The electrical conduction of the contour, as different from the spatial case, has its effect on the induced magnetic field, only (see, [2]).

3. Function of current

Three planes are definable while doing the problem (13) – (15):

• Base plane $z = x + iy$ of contour enveloped by the flow L_z,

• Induced plane associated with the base one [2] $\zeta = \xi + i\eta$:

$$x = \xi\sqrt{\lambda^2+S}, \quad y = \lambda\eta \qquad (17)$$

L_ζ – map of the contour L_z during the transformation (17),

• τ-plane with the polar coordinates $\tau = r\exp i\theta$, which is associated with the induced plane ζ via conformal mapping

$$\zeta = k\tau + k_0 + \sum_{n=1}^{\infty}\frac{k_n}{\tau^n}, \quad k > 0 \qquad (18)$$

of the exterior of the contour L_ζ on the exterior of the circle $r = 1$. As in [2]

$$\varphi = \varphi_1 + \varphi_2 \qquad (19)$$

where φ_1 – the velocity potential in the problem of the contour L_ζ which is enveloped in ζ-plane by the flow with the velocity $\mathbf{U}$ at infinity (see. (15)):

$$\mathbf{U} = \lambda^2(\sqrt{\lambda^2+S} + i\lambda\sin\alpha) \qquad (20)$$

so that in ζ-plane the function φ_1 is the resolution of the problem:

$$\nabla^2\varphi_1 = 0; \quad \left.\frac{\partial\varphi_1}{\partial n}\right|_{L_\zeta} = 0, \quad \left.\frac{\partial\varphi_1}{\partial\xi} + i\frac{\partial\varphi_1}{\partial\eta}\right|_{\infty} = U \qquad (21)$$

φ_2 – the Neumann problem solution inside the exterior of the contour L_ζ (see. (13)):

$$\nabla^2\varphi_2 = 0;\quad \left.\frac{\partial\varphi_2}{\partial n}\right|_{L_\zeta} = -\lambda S\cos(n,\eta)\sin\alpha\Big|_{L_\zeta}. \qquad (22)$$

The Laplacian in (21) and (22)) has the form of (14).

Let $\tau = \tau(\zeta)$ is the conformal mapping of the exterior of the circle $|\tau| = 1$ in τ-plane on the exterior of the contour L_ζ in the plane ζ. $\tau = \tau(\zeta)$ – the function which is inverse to the function (18). The complex potential of the flow passing round the contour L_ζ in ζ-plane at the velocity $\mathbf{U}$ at infinity [4] $w(\zeta) = k\overline{U}\tau(\zeta) + \dfrac{kU}{\tau(\zeta)}$, $k > 0$, or from (21) $\varphi_1 = \operatorname{Re} w(\zeta)$, or

$$\varphi_1 = k\operatorname{Re}\left[\overline{U}\tau(\zeta) + \frac{kU}{\tau(\zeta)}\right]. \qquad (23)$$

Let us formulate the problem (22) in τ- plane. Let $H = |\mathrm{d}\,\zeta/\mathrm{d}\,\tau|$. Then the condition (13) in τ-plane has the form $\left.\dfrac{1}{H}\dfrac{\partial\varphi_2}{\partial r}\right|_{r=1} = -\left.\dfrac{\lambda\sin\alpha}{H}\dfrac{\partial\eta}{\partial r}\right|_{r=1}$, and (22) is the Neumann problem for the exterior of the $|\,\zeta\,| = 1$:

$$\begin{gathered}\nabla^2\varphi_2 = 0,\\ \nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2},\\ \left.\frac{1}{H}\frac{\partial\varphi_2}{\partial r}\right|_{r=1} = -\left.\frac{\lambda S}{H}\frac{\partial\eta}{\partial r}\right|_{r=1}\end{gathered} \qquad (24)$$

$\eta(r,\theta) = \operatorname{Im}\zeta(r\exp(i\theta))$, where the function $\zeta(\tau)$ is determined in (18). This is why

$$\begin{aligned}\frac{\partial\eta}{\partial r} &= \operatorname{Im}\left(\frac{\mathrm{d}\,\zeta}{\mathrm{d}\,\tau}\frac{\partial\tau}{\partial r}\right) = \operatorname{Im}\left(\mathrm{e}^{\,i\theta}\frac{\mathrm{d}\,\zeta}{\mathrm{d}\,\tau}\right)\\ &= \operatorname{Im}\left[\left(k - \sum_{n=1}^{\infty}\frac{nk_n}{\tau^{n+1}}\right)\mathrm{e}^{\,i\theta}\right]\\ &= \operatorname{Im}\left(k\mathrm{e}^{\,i\theta} - \sum_{n=1}^{\infty}\frac{nk_n}{r^{n+1}}\mathrm{e}^{\,-in\theta}\right)\end{aligned}$$

the conditions of the problem (24) too, assume the form (as $k > 0$ – real)

$$\begin{gathered}\nabla^2\varphi_2 = 0,\\ \left.\frac{\partial\varphi_2}{\partial r}\right|_{r=1} = -\lambda S\Bigg(k\sin\theta\\ -\operatorname{Im}\sum_{n=1}^{\infty} nk_n\mathrm{e}^{\,-in\theta}\Bigg).\end{gathered} \qquad (25)$$

∇^2 – the Laplacian in the polar coordinates r,θ. That is why

$$\varphi_2 = -\lambda S\left(\frac{C\sin\theta}{r} + \operatorname{Im}\sum_{n=1}^{\infty}\frac{C_n}{r^n}\mathrm{e}^{\,-in\theta}\right)\sin\alpha$$

The substitution in the boundary condition (25) comes out with $C = -k$, $C_n = k_n$, and

$$\varphi_2 = \lambda S\left(\frac{k\sin\theta}{r} - \operatorname{Im}\sum_{n=1}^{\infty}\frac{k_n}{r^n}\mathrm{e}^{\,in\theta}\right)\sin\alpha. \qquad (26)$$

In this way, the field velocity and induced current are determined with the formulae (19), (23) and (26)when the conductive fluid flow passes round the arbitrary contour L_z in magnetic field in the *inductive approximation* the complex number U having the form of (16), (20), and numbers k and k_n being determinable according to the one-to-one and conformal mappings (18) of the exterior of the contour L_ζ on the exterior of the unity circle in τ-plane. The contour L_ζ is produced from the main contour L_z via the affine transformation (17). The parameters λ and S are introduced in reference [2]. In order to construct the expressions(16) it is necessary in (23) and (26) to go from the polar coordinates r, θ in τ-plane to the coordinates ξ, η in ζ-plane according to the mapping (18), and then go from the coordinates ξ, η to the coordinates x, y in (17).

4. Turbulent force action

The intricate procedure of going from the variables r,θ to the variables x,y is not needed to determine the effect of flow on the contour, since the integration should be made in τ- plane. For the turbulent [2] component of the force $\mathbf{R}_1$ with which the flow impacts the contour, we obtain ($V = v_\infty$)

$$\mathbf{R}_1 = \frac{X_1 - iY_1}{\rho V^2 L_*} = i\int_{L_z}\varphi\mathrm{d}\,\overline{z} - \lambda^2\Omega\mathrm{e}^{\,i\alpha} \qquad (27)$$

Considering (17) and (19), we find by going to ζ-plane that

$$\begin{aligned}i\int_{L_z}\varphi\mathrm{d}\,\overline{z} &= i\int_{L_z}(\varphi_1+\varphi_2)\mathrm{d}\,\overline{z}\\ &= i\int_{L_\zeta}(\varphi_1+\varphi_2)(\sqrt{\lambda^2+S}\mathrm{d}\,\xi - i\mathrm{d}\,\eta).\end{aligned} \qquad (28)$$

On the contour L_ζ during the transition to τ-plane $r = 1$, $\mathrm{d}\,r = 0$, and from (18)

$$\begin{aligned}\left.\mathrm{d}\,\xi\right|_{L_\zeta} &= -\operatorname{Im}\left(k\mathrm{e}^{\,i\theta} - \sum_{n=1}^{\infty} nk_n\,e^{-in\theta}\right)\mathrm{d}\,\theta,\\ \left.\mathrm{d}\,\eta\right|_{L_\zeta} &= \operatorname{Re}\left(k\mathrm{e}^{\,i\theta} - \sum_{n=1}^{\infty} nk_n\mathrm{e}^{\,-in\theta}\right)\mathrm{d}\,\theta,\\ \left.\mathrm{d}\,\zeta\right|_{L_\zeta} &= i\left(k\mathrm{e}^{\,i\theta} - \sum_{n=1}^{\infty} nk_n\mathrm{e}^{\,-in\theta}\right)\mathrm{d}\,\theta\end{aligned} \qquad (29)$$

to the apex L_ζ $\tau = \exp(i\theta)$, and from (23), $\varphi_1\Big|_{L_\zeta} = k\mathrm{Re}\left(\overline{U}\mathrm{e}^{i\theta} + U\mathrm{e}^{-i\theta}\right)$. That is why

$$\int\limits_{L_\zeta} \varphi_1 \mathrm{d}\,\zeta = ki\int\limits_0^{2\pi}\left(\overline{U}\mathrm{e}^{i\theta} + U\mathrm{e}^{-i\theta}\right) \times \left(k\mathrm{e}^{i\theta} - \sum_{n=1}^{\infty} nk_n\mathrm{e}^{-in\theta}\right)\mathrm{d}\,\theta$$

$$= ki\int\limits_0^{2\pi}\left(\overline{U}\mathrm{e}^{i\theta} + U\mathrm{e}^{-i\theta}\right)\left(k\mathrm{e}^{i\theta} - k_1\mathrm{e}^{-i\theta}\right)\mathrm{d}\,\theta$$

$$\Rightarrow \int\limits_{L_\zeta} \varphi_1(\mathrm{d}\,\xi + i\mathrm{d}\,\eta) = 2k\pi i(k\overline{U} - k_1 U)$$

imaginary part on $i\sqrt{\lambda^2+S}$ and on λ:

$$\int\limits_{L_\zeta} \varphi_1 \mathrm{d}\,\overline{z} = 2k\pi[\lambda\mathrm{Re}\,(kU - k_1\overline{U}) - i\sqrt{\lambda^2+S}\mathrm{Im}\,(kU - k_1\overline{U})]. \quad (30)$$

To compute in (28) the integral of φ_2 we shall consider (26), taking into account what we have also (see. (18)), for $r^{-n}\mathrm{e}^{-in\theta} = \tau^n$, $\sin\theta = -\mathrm{Im}\,(\mathrm{e}^{-i\theta})$, φ_2:

$$\varphi_2 = \lambda S\mathrm{Im}\left[k\left(\tau - \frac{1}{\tau}\right) + k_0 - \zeta(\tau)\right]\sin\alpha. \quad (31)$$

On the contour the apexes $\tau = \mathrm{e}^{i\theta}$, $\zeta = \xi + i\eta$ and

$$\int\limits_{L_\zeta} \varphi_2 \mathrm{d}\,\zeta = \lambda S\left[2k\int\limits_0^{2\pi}\sin\theta \mathrm{d}\,\zeta - \int\limits_{L_\zeta}\eta(\mathrm{d}\,\xi + i\mathrm{d}\,\eta)\right]\sin\alpha.$$

We find that $\int\limits_{L_\zeta}\eta \mathrm{d}\,\xi = \int\limits_{L_z}\frac{y\mathrm{d}\,x}{\lambda\sqrt{\lambda^2+S}} = -\frac{\Omega}{\lambda\sqrt{\lambda^2+S}}$ (see. (17)), Ω – the dimensionless area of the contour L_z. Now (see. (29)),

$$2\int\limits_0^{2\pi}\sin\theta \mathrm{d}\,\zeta = \int\limits_0^{2\pi}(\mathrm{e}^{i\theta}\mathrm{e}^{-i\theta})(k\mathrm{e}^{i\theta}k_1\mathrm{e}^{-i\theta})\mathrm{d}\,\theta = -2\pi(k+k_1)$$

$$\Rightarrow \int\limits_{L_\zeta}\varphi_2 \mathrm{d}\,\zeta = \lambda S\left[\frac{\Omega}{\lambda\sqrt{\lambda^2+S}} - 2\pi(k+k_1)\right]\sin\alpha.$$

By separating the real and imaginary parts (see. (28)), we find that

$$i\int\limits_{L_\zeta}\varphi_2 \mathrm{d}\,\overline{z} = iS\Omega\sin\alpha + 2k\pi\lambda S\Big\{\lambda\mathrm{Re}\,[i(k+k_1)] - i\sqrt{\lambda^2+S}\mathrm{Im}\,[i(k+k_1)]\Big\}\sin\alpha]. \quad (32)$$

The term $\mathrm{Re}\,(ik)$ (k – the real number) which is zero is preserved for the sake of the symmetry. We shall introduce the last integral and integral (30) into (28) and substitute the thus-obtained integral in (27). Upon transition to complex conjugates, we shall define the turbulent part of the force of the flow acting on the contour in magnetic field:

$$\mathbf{R}_1 = \frac{X_1 - iY_1}{\rho V^2 L_*} = \lambda\mathrm{Re}\,w + i\sqrt{\lambda^2+S}\mathrm{Im}\,w - \Omega[\lambda^2\cos\alpha + i(\lambda^2+S)\sin\alpha], \quad (33)$$

$$w = 2k\pi(kW - k_1\overline{W}), \quad \overline{W} = U + i\lambda S\sin\alpha \quad (34)$$

5. Euler-d'Alambert paradox

When nonconductive fluid flows round the contour $S = 0$,the Stewart number $U = \lambda^3\mathrm{e}^{i\alpha}$ and from (33)

$$\mathbf{R}_1 = \frac{X_1 - iY_1}{\rho\lambda^2 V^2 L_*} = 2\lambda k\pi(\lambda k\mathrm{e}^{i\alpha} - \lambda k_1\mathrm{e}^{-i\alpha}) - \Omega\mathrm{e}^{i\alpha}. \quad (35)$$

The numbers k and k_1 are determined by the mapping (18) of the induced contour L_ζ on the exterior of the unity circle. Yet, at $S = 0$ the plane $\zeta = \xi + i\eta$ is not needed: one has to proceed from the map of the exterior of the contour L_z on the exterior of the unity circle. We shall obtain this map from (18), if we take into account the union (17) ξ and η with x and y at $S = 0$. Then from (18) it follows that

$$\zeta = \frac{x+iy}{\lambda} = k\tau + k_0 + \sum_{n=1}^{\infty}\frac{k_n}{\tau^n} \Rightarrow z = \lambda k\tau + \lambda k_0 + \sum_{n=1}^{\infty}\frac{\lambda k_n}{\tau^n}.$$

The conformal mapping of the exterior of the contour L_z on the exterior of the circle with a motionless infinitely distant point is constructed directly:

$$z = q\tau + q_0 + \sum_{n=1}^{\infty}\frac{q_n}{\tau^n}. \quad (36)$$

Let us compare the series (36) with the preceding one. Due to the straightforwardness of the mapping we find $\lambda k = q$, $\lambda k_n = q_n$, $n = 0, 1, 2, \ldots$, and from (35) ($\lambda = \sqrt{\kappa L_*^2/\nu}$)

$$\mathbf{R}_1 = \frac{X_1 - iY_1}{\rho\kappa V L_*^2} = 2\pi q\Big(q\mathrm{e}^{i\alpha} - q_1\mathrm{e}^{-i\alpha}\Big) - \Omega\mathrm{e}^{i\alpha}. \quad (37)$$

κ – the turbulent resistance coefficient [2]. The numbers q and q_1 are determined in (36). This is the turbulent part of the nonconductive fluid flow force acting on the arbitrary contour L_z. The formula (37) gets rid of the Euler-d'Alambert paradox in the plane case.

6. Hydrodynamic moment turbulent part

We shall now deal with the turbulent part of the moment $\mathbf{M}_1'$ [2]:

$$\mathbf{M}_1 = \frac{\mathbf{M}_1'}{\rho V^2 L_*^2} = -\int\limits_{L_z} (\varphi_1 + \varphi_2)(x\,\mathrm{d}\,x + y\,\mathrm{d}\,y) = \mathbf{M}_{11} + \mathbf{M}_{12}. \quad (38)$$

Considering (17), while going to ζ-plane, we find that

$$\mathbf{M}_{1j} = -\lambda^2 \int\limits_{L_\zeta} \varphi_j(\xi\,\mathrm{d}\,\xi + \eta\,\mathrm{d}\,\eta) - S\int\limits_{L_\zeta} \varphi_j \xi\,\mathrm{d}\,\xi, \quad j = 1, 2. \quad (39)$$

The integrals (39) are calculated by transiting to τ-plane of the unity circle. To determine the incorporated differential forms in the unambiguous relation $\xi\mathrm{d}\,\xi + \eta\mathrm{d}\,\eta = \mathrm{Re}\,(\overline{\zeta}\mathrm{d}\,\zeta)$, $\xi\mathrm{d}\,\xi = \frac{1}{2}\mathrm{Re}\,[(\zeta + \overline{\zeta})\, d\zeta]$ we shall use the expressions (18) and (29) for the contour L_ζ, i.e. at $\tau = \mathrm{e}^{i\theta}$. Then

$$\xi\mathrm{d}\,\xi\Big|_{L_\zeta} = -\frac{1}{2}\mathrm{Im}\Bigg\{k^2\mathrm{e}^{2i\theta} - k\sum_{m=0}^{\infty} k_m\left[(m-1)\mathrm{e}^{-i(m-1)\theta} + m\mathrm{e}^{-i(m+1)\theta}\right] + k\sum_{m=0}^{\infty} \overline{k}_m\mathrm{e}^{i(m+1)\theta} \sum_{m=0,n=1}^{\infty} nk_n\left[k_m\mathrm{e}^{-i(m+n)\theta} + \overline{k}_m\mathrm{e}^{i(m-n)\theta}\right]\Bigg\}\Bigg|_{L_\zeta} \mathrm{d}\,\theta, \quad (40)$$

$$\xi\mathrm{d}\,\xi + \eta\mathrm{d}\,\eta\Big|_{L_\zeta} = \mathrm{Im}\left[\sum_{m=1}^{\infty} mk_m\mathrm{e}^{-i(m+1)\theta} - k\sum_{m=0}^{\infty}\overline{k}_m\mathrm{e}^{i(m+1)\theta}\right] + \mathrm{Im}\left[\sum_{m=0,n=1}^{\infty} nk_m\overline{k}_m\mathrm{e}^{i(m-n)\theta}\right]_{L_\zeta} \mathrm{d}\,\theta.$$

In the same contour from (23) it follows that

$$\varphi_1\Big|_{L_\zeta} = k\left(\overline{U}\mathrm{e}^{i\theta} + U\mathrm{e}^{-i\theta}\right) \quad (41)$$

and in (39) (see. (40))

$$\int\limits_{L_\zeta} \varphi_1(\xi\mathrm{d}\,\xi + \eta\mathrm{d}\,\eta) = 2k\pi\mathrm{Im}\left[-k\overline{k}_0 + \sum_{n=1}^{\infty} nk_n\left(\overline{k}_{n-1}\overline{U} + \overline{k}_{n+1}U\right)\right].$$

The integral in (39), containing $\varphi\xi\mathrm{d}\,\xi$, and in consideration of (40), is computed similarly. In the final run ($\mathrm{Im}\,z = -\mathrm{Im}\,\overline{z}$)

$$M_{11} = k\pi\mathrm{Im}\Bigg\{S\left[kk_0U(kk_2 + k_0k_1)\overline{U}\right] + (2\lambda^2 + S)\Bigg[k\overline{k}_0U + \sum_{n=1}^{\infty} n\overline{k}_n\left(k_{n-1}U + k_{n+1}\overline{U}\right)\Bigg]\Bigg\}. \quad (42)$$

To calculate M_{12} we find from (31) that

$$\varphi_1\Big|_{L_\zeta} = \lambda S[2k\sin\theta + \mathrm{Im}\,k_0 - \eta]\Big|_{L_\zeta}. \quad (43)$$

By introducing (43) in (29), we make the calculations using (17) and the Green formula,

$$\int\limits_{L_\zeta} \xi\eta\mathrm{d}\,\xi = \frac{1}{\lambda^2(\lambda^2+S)}\int\limits_{L_z} xy\mathrm{d}\,x = -\frac{1}{\lambda^2(\lambda^2+S)}\int\limits_{\Omega} x\mathrm{d}\,x\mathrm{d}\,y = 0$$

since the double integral is the static moment of the area Ω of the contour L_z relative to the axis Oy, traversing the centre-of-gravity of the area. So, while substituting (43) in (39) it may just suffice to consider the integrals of the function $k\lambda S[-i\exp(i\theta) + i\exp(-i\theta)]\sin\alpha$. By comparing this function with (41), we can see that in order to obtain $\mathbf{M}_{12}$ in the expression $\mathbf{M}_{11}$ for U it would be just enough to substitute the number for $i\lambda S\sin\alpha$. To obtain the sought-after sum in (38) $\mathbf{M}_{11} + \mathbf{M}_{12}$ found in (42) it might be simpler to substitute U for (see. (34) and so on considering (20)):

$$w = U + i\lambda S\sin\alpha = \lambda\sqrt{\lambda^2+S}(\lambda\sin\alpha + i\sqrt{\lambda^2+S}\sin\alpha) \quad (44)$$

$$\mathbf{M}_1 = \frac{\mathbf{M}_1'}{\rho V^2 L_*^2} = k\pi\mathrm{Im}\Bigg\{\left[kk_0W - (kk_2 + k_0k_1)\overline{W}\right] + (2\lambda^2 + S) \times \Bigg[k\overline{k}_0W + \sum_{n=1}^{\infty} n\overline{k}_n\left(k_{n-1}W + k_{n+1}\overline{W}\right)\Bigg]\Bigg\}. \quad (45)$$

$S = 0$ from (20) and (44) $W = \lambda^3\mathrm{e}^{i\alpha}$, and from (45) in similarity to the formula (37) for the force $\mathbf{R}_1$, we obtain the expression for the turbulent part of the moment that would collapse the arbitrary contour in the flow of the ideal nonconductive fluid L_z

$$\mathbf{M}_1 = \frac{\mathbf{M}_1'}{\rho\kappa V L_*^3} = 2\pi q \mathrm{Im}\left[q\overline{q}_0 \mathrm{e}^{\,i\alpha} + \sum_{n=1}^{\infty}\left(q_{n-1}\mathrm{e}^{\,i\alpha} + q_{n+1}\mathrm{e}^{\,i\alpha}\right)\right]. \quad (46)$$

7. Integral representations

Let us construct the integral representations of the series (45) and (46). Let $\tau = \tau(\zeta)$ – be the conformal mapping of the exterior of the circle in τ-plane on the exterior of the contour L_ζ. $\tau(\zeta)$ – the function which is inverse to the function (18).

Since on the contour L_ζ $\tau(\zeta) = \mathrm{e}^{\,i\theta}$, then according to (18) and (29)

$$\left.\frac{\overline{\zeta}\mathrm{d}\,\zeta}{\tau(\zeta)}\right|_{L_\zeta} = i \lim_{r\to 1+0}\left(k\mathrm{e}^{\,-i\theta} + \sum_{n=0}^{\infty}\frac{\overline{k}_n}{r^n}\mathrm{e}^{\,in\theta}\right) \times \left(k - \sum_{m=1}^{\infty}\frac{mk_m}{r^m}\mathrm{e}^{\,-i(m+1)\theta}\right)\mathrm{d}\,\theta$$
$$= i \lim_{r\to 1+0}\left(k\mathrm{e}^{\,-i\theta} + \sum_{n=0}^{\infty}\frac{\overline{k}_n}{r^n}\mathrm{e}^{\,in\theta}\right) \times \left(k - \sum_{m=2}^{\infty}\frac{m-1}{r^{m-1}}k_{m-1}\mathrm{e}^{\,-im\theta}\right)d\theta$$
$$= i \lim_{r\to 1+0}\left[k^2\mathrm{e}^{\,-i\theta} + k\sum_{n=0}^{\infty}\frac{\overline{k}_n}{r^n}\mathrm{e}^{\,in\theta} - k\sum_{m=2}^{\infty}\frac{m-1}{r^{m-1}}k_{m-1}\mathrm{e}^{\,i(m+1)\theta} - \sum_{m=2,n=0}^{\infty}\frac{m-1}{r^{m+n-1}}\overline{k}_n k_{m-1}\mathrm{e}^{\,i(n-m)\theta}\right]\mathrm{d}\,\theta.$$

Under the limiting sign the series converge at any value of $r > 1$ and they can be integrated over θ from zero to 2π. In ζ-plane, this kind of integration corresponds to the integration over the contour L_ζ. Which is why

$$\mathrm{Im}\left(k\overline{k}_0 + \sum_{m=1}^{\infty} m\overline{k}_m k_{m+1}\right) = \mathrm{Im}\left[\frac{1}{2\pi i}\int_{L_\zeta}\frac{\overline{\zeta}\mathrm{d}\,\zeta}{\tau(\zeta)}\right],$$
$$\mathrm{Im}\sum_{m=1}^{\infty}\overline{k}_m k_{m-1} = \mathrm{Im}\left[\frac{1}{2\pi i}\int_{L_\zeta}\overline{\zeta}\tau(\zeta)\mathrm{d}\,\zeta\right].$$

The second integral representation is constructed similarly. The sums in the formula (46) have integral representations of the same kind after replacement of ζ with z considering (36). However, there is a major difference between the sums (or their integral representations) in (46) and (45). In the absence of the field in (46) the sums depend on the contour properties for their determination. If the flow is conductive and magnetic field applied, then the similitude (17) deforms the contour and the sums in (45) are determined not only by the properties of the contour, but also by the game of the turbulent resistance and applied magnetic field. Note that the series, for which the integral representations are constructed, can diverge, and those series can be summed by using one of the well-known methods for divergent series summation(for example, the Abel method) [6].

8. Effect of inertia

Let us construct the integral representations of the inertial [2] effect of the flow on the contour: the moment $\mathbf{M}_2'$ and the force $\mathbf{R}_2'$, as referred to in the unit length. In the expressions we employ [2] The Blazius-Chaplygin formulae [4]:

$$\mathbf{M}_2 = \frac{\mathbf{M}_2'}{\rho V^2 L_*^2} = -\frac{1}{2}\mathrm{Re}\int_{L_z}\overline{v}z\mathrm{d}\,z,$$
$$\mathbf{R}_2 = \frac{X_2 i Y_2}{\rho V^2 L_*} = \frac{1}{2}i\int_{L_z}\overline{v}^2\mathrm{d}\,z. \quad (47)$$

Here according to (16) and (17) the complex velocity is

$$\overline{v} = v_x - iv_y = \frac{1}{\lambda^2}\frac{\partial\varphi}{\partial x} - \frac{i}{\lambda^2+S}\left(\frac{\partial\varphi}{\partial y} + S\sin\alpha\right) = \frac{1}{\lambda^2\sqrt{\lambda^2+S}}\frac{\partial\varphi}{\partial\xi} - \frac{i}{\lambda^2+S}\left(\frac{1}{\lambda}\frac{\partial\varphi}{\partial\eta} + S\sin\alpha\right).$$

Let us introduce a function which is equal to the velocity potential difference at L_ζ:

$$F = f + ig,\quad f = \left.\frac{\partial\varphi}{\partial\eta}\right|_{L_\zeta} + \lambda S\sin\alpha,\quad g = \left.\frac{\partial\varphi}{\partial\xi}\right|_{L_\zeta}. \quad (48)$$

Then

$$\left.\overline{v}\right|_{L_\zeta} = \frac{g}{\lambda^2\sqrt{\lambda^2+S}} - \frac{if}{\lambda(\lambda^2+S)},$$
$$\left.\overline{v}^2\right|_{L_\zeta} = \frac{g^2}{\lambda^2(\lambda^2+S)} - \frac{f^2}{\lambda^2(\lambda^2+S)} - \frac{2ifg}{\lambda^3(\lambda^2+S)^{3/2}}. \quad (49)$$

The idea of transformation of integrals (47) lies in extraction in them of such part that at $S = 0$ – in the absence of a field is transformed into the classical expressions for the moment and force. We omit those transformations that are based on the use of the formulae (17),(48),(49):

$$\mathbf{M}_2 = \frac{\mathbf{M}_2'}{\rho V^2 L_*^2} = \mathrm{Re}\int\limits_{L_\zeta} \frac{F^2\zeta\mathrm{d}\,\zeta}{2\lambda^2(\lambda^2+S)}$$

$$+S\int\limits_{L_\zeta}\frac{\lambda^2(\,\mathrm{Re}\,F^2)\eta\mathrm{d}\,\eta(\lambda^2+S)(\mathrm{Im}\,F)^2\xi\mathrm{d}\,\xi}{2\lambda^4(\lambda^2+S)^2}, \quad (50)$$

$$\mathbf{R}_2 = \frac{X_2 - iY_2}{\rho V^2 L_*}$$

$$= \int\limits_{L_\zeta}\frac{\lambda\mathrm{Im}\,(F^2\mathrm{d}\,\zeta) - i\sqrt{\lambda^2+S}\mathrm{Re}\,(F^2 d\zeta)}{2\lambda^2(\lambda^2+S)^2} + S$$

$$\times\int\limits_{L_\zeta}(\lambda\mathrm{Im}\,(F^2)\mathrm{d}\,\xi - (\lambda\mathrm{d}\,\eta - i\sqrt{\lambda^2+S}\mathrm{d}\,\xi)\mathrm{Im}\,F^2)$$

$$/(2\lambda^4(\lambda^2+S)^2)d\xi. \quad (51)$$

We shall find the derived functions φ included in (48) as defined in (19). By differentiating η the relation (23), over we obtain

$$\left.\frac{\partial\varphi_1}{\partial\eta}\right|_{L_\zeta} = k\mathrm{Re}\,\left[\overline{U}\frac{\partial\tau}{\partial\zeta}\frac{\partial\zeta}{\partial\eta} - \frac{U}{\tau^2}\frac{\partial\tau}{\partial\zeta}\frac{\partial\zeta}{\partial\eta}\right]_{L_\zeta}$$

but $\tau|_{L_\zeta} = \mathrm{e}^{\,i\theta}$, $\zeta = \xi + i\eta$, $\partial\zeta/\partial\eta = i$ and

$$\left.\frac{\partial\varphi_1}{\partial\eta}\right|_{L_\zeta} = k\mathrm{Re}\,\left[i\left(\overline{U} - U\mathrm{e}^{\,2i\theta}\right)\frac{\mathrm{d}\,\tau}{\mathrm{d}\,\zeta}\right]_{r=1},$$
$$\left.\frac{\partial\varphi_1}{\partial\xi}\right|_{L_\zeta} = k\mathrm{Im}\,\left[i\left(\overline{U} - U\mathrm{e}^{\,2i\theta}\right)\frac{\mathrm{d}\,\tau}{\mathrm{d}\,\zeta}\right]_{r=1} \quad (52)$$

since $\mathrm{Re}\,z = \mathrm{Im}\,(iz)$, $\partial\zeta/\partial\xi = 1$. The derived functions φ_2 are determined similarly from (31):

$$\left.\frac{\partial\varphi_2}{\partial\eta}\right|_{L_\zeta} + S\sin\alpha = \lambda kS\mathrm{Re}\,\left[\left(1+\mathrm{e}^{\,2i\theta}\right)\frac{\mathrm{d}\,\tau}{\mathrm{d}\,\zeta}\right]_{r=1}\sin\alpha,$$
$$\left.\frac{\partial\varphi_2}{\partial\xi}\right|_{L_\zeta} = \lambda kS\mathrm{Im}\,\left[\left(1+\mathrm{e}^{\,2i\theta}\right)\frac{\mathrm{d}\,\tau}{\mathrm{d}\,\zeta}\right]_{r=1}\sin\alpha.$$

By adding the derivatives of the same kind in (52) and in the last formulae, and by substituting the sums in (48) ($\varphi = \varphi_1 + \varphi_2$), we shall obtain f and g, and then

$$F = f + ig = k\Big[\lambda S\sin\alpha + iU$$
$$\left.+(\lambda S\sin\alpha - iU)\mathrm{e}^{\,2i\theta}\Big]\frac{d\tau}{d\zeta}\right|_{r=1} \quad (53)$$

$\mathrm{d}\,\tau/\mathrm{d}\,\zeta$ – the out-of-the-circle $|\tau| < 1$ analytical function of the complex variable ζ, which is not zero and is definable according to (18):

$$\frac{\mathrm{d}\,\tau}{\mathrm{d}\,\zeta} = \left(k - \sum_{n=1}^{\infty}\frac{nk_n}{\tau^{n+1}}\right)^{-1}.$$

Its expansion at infinity has not the term containing τ^{-1}:

$$\frac{\mathrm{d}\,\tau}{\mathrm{d}\,\zeta} = \frac{1}{k} + \frac{k_1}{k^2\tau^2} + \frac{2k_2}{k^2\tau^3} + O\left(\frac{1}{\tau^4}\right). \quad (54)$$

Let us calculate the integral containing $F^2\mathrm{d}\,\zeta$ in (51). For the sake of simplicity, we shall re-write the expression (53) taking into account (44) as ($\mathrm{e}^{\,-2i\tau} = \tau^{-2}$ at $r=1$, $W = U + i\lambda S\sin\alpha$)

$$F = ik\left(\overline{W} - \frac{W}{\tau^2}\right)\left.\frac{\mathrm{d}\,\tau}{\mathrm{d}\,\zeta}\right|_{r=1} \quad (55)$$

Then

$$\int\limits_{L_\zeta} F^2\mathrm{d}\,\zeta = \int\limits_{|\tau|=1} F^2\frac{\mathrm{d}\,\zeta}{\mathrm{d}\,\tau}\mathrm{d}\,\tau$$
$$= -k^2\int\limits_{|\tau|=1}\left(\overline{W} - \frac{W}{\tau^2}\right)^2\frac{\mathrm{d}\,\tau}{\mathrm{d}\,\zeta}\mathrm{d}\,\tau.$$

The integrand function is here analytic out of the circle $|\,\tau\,| = 1$ in τ-plane, including the infinitely distant point. That is why the integral is proportional to the residue of the function F^2 relative to the infinitely distant point $\tau = \infty$. Due to (54) this residue is zero. In this way

$$\int\limits_{L_\zeta} F^2\mathrm{d}\,\zeta = 0. \quad (56)$$

In the absence of magnetic field, the relation (56) is known as the Euler-d'Alambert paradox. Included in (50) is

$$\mathrm{Re}\int\limits_{L_\zeta}\zeta F^2\mathrm{d}\,\zeta = -k^2\mathrm{Re}\int\limits_{|\tau|=1}\zeta\left(\overline{W} - \frac{W}{\tau^2}\right)^2\frac{\mathrm{d}\,\tau}{\mathrm{d}\,\zeta}\mathrm{d}\,\tau.$$

Let us determine the residue of the integrand relative to the infinitely distant point in τ-plane. We shall employ (18) and (54):

$$\zeta\left(\overline{W} - \frac{W}{\tau^2}\right)\frac{d\tau}{d\zeta} = \left(k\tau + k_0 + \frac{k_1}{\tau}\right)\times$$
$$\left(\overline{W}^2 - \frac{2\overline{W}W}{\tau^2}\right)\left(\frac{1}{k} + \frac{1}{k^2\tau^2}\right) + \ldots = \overline{W}^2\tau +$$
$$\frac{k_0\overline{W}^2}{k} + \frac{2\overline{W}}{\tau}\left(\frac{k_1}{k}\overline{W} - W\right) + O\left(\frac{1}{\tau^2}\right).$$

Consequently, according to the theorem on residues

$$\mathrm{Re}\int\limits_{L_\zeta}\zeta F^2\mathrm{d}\,\zeta=4\pi k^2\mathrm{Im}\,\left[\overline{W}(c\overline{W}-W)\right] \qquad (57)$$

where $c=k_1/k$. Let us introduce the integrals (56) and (57) in the formulae (50) and (51). Then we shall derive the expressions for the inertial part of the conductive fluid flow impact on the contour in magnetic field:

$$\mathbf{R}_2=\frac{X_2-iY_2}{\rho V^2L_*}=S\int\limits_0^{2\pi}\frac{\lambda\mathrm{Im}\,(F^2)\mathrm{d}\,\xi}{2\lambda^4(\lambda^2+S)^2}-S\int\limits_0^{2\pi}\frac{(\mathrm{Im}\,F)^2(\lambda\mathrm{d}\,\eta+i\sqrt{\lambda^2+S}\mathrm{d}\,\xi)}{2\lambda^4(\lambda^2+S)^2}, \qquad (58)$$

$$\mathbf{M}_2=\frac{\mathbf{M}_2'}{\rho V^2L_*^2}=\frac{2\pi k^2\mathrm{Im}\,[\overline{W}(c\overline{W}-W)]}{\lambda^2(\lambda^2+S)}-S\frac{\lambda^2(I_\xi-I_\eta)+SI_\xi}{\lambda^4(\lambda^2+S)^2},$$

$$c=\frac{k_1}{k},\quad I_\xi=\int\limits_0^{2\pi}(\mathrm{Im}\,F)^2\xi\mathrm{d}\,\xi,\quad I_\eta=\int\limits_0^{2\pi}(\mathrm{Re}\,F)^2\eta\mathrm{d}\,\eta$$

where the integration is made over the azimuth θ of the polar coordinates in τ- plane. The function $F(\theta)$ is determined in (55). The differential forms $\mathrm{d}\,\xi$ and $\mathrm{d}\,\eta$ are expressed in terms of θ and $\mathrm{d}\,\theta$ in (29), and the forms $\xi\mathrm{d}\,\xi$ and $\eta\mathrm{d}\,\eta$ in terms of (40). At $S=0$ from the formulae (58) $\mathbf{M}_2=2\lambda k\mathrm{Im}\,(\lambda k_1\mathrm{e}^{-2i\alpha})=2\pi q\mathrm{Im}\,(q_1\mathrm{e}^{-2i\alpha})$. This is the well-known result of the classical hydrodynamics [4].

9. Ellipse. Preliminary formulae

Finishing up the description of the process of flow of the non-viscous conductive fluid round bodies in magnetic field in the inductive approximation, as started in [2], we shall now define the effect of the flow on the ellipse in magnetic field in the plane of its contour. Let $a'\geq b'\geq c'\geq 0$ of the semi-axis of the ellipse. Its canonical equation in the dimensionless variables [2] with the length scale $L_*=a'$ has the form

$$\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}+\frac{z_1^2}{c^2}=1 \qquad (59)$$

For the sake of the symmetry, we preserve the designation a ($a=1$) of the larger semi-axis of the ellipse (59). Let the vector $\mathbf{B}^0$ of the applied magnetic field form the angle β with the larger axis of the ellipse, α being the angle between the vector $\mathbf{V}^0$ of the running flow velocity and vector $\mathbf{B}^0$. According to [2] the axis Ox in the flow plane $z=x+iy$ should be directed along the vector $\mathbf{B}^0$. For this reason, in the ellipse equation one has to go from the axes Ox_1y_1 to the axes Oxy, which are associated with applied magnetic field. For this purpose, the plane $z_1=x_1+iy_1$ should be turned at the angle $(-\beta)$ (so that $\arg z_1=\arg z+\beta$). Therefore $z_1=ze^{i\beta}$, or, in more detail, $x_1+iy_1=(x+iy)(\cos\beta+i\sin\beta)$, whereform $x_1=x\cos\beta-y\sin\beta$, $y_1=x\sin\beta+y\cos\beta$, the equation (59) assumes the form

$$L_z:\quad\frac{(x\cos\beta-y\sin\beta)^2}{a^2}+\frac{(x\sin\beta+y\cos\beta)^2}{b^2}=1.$$

In accordance with (17) the ellipse L_z should undergo the affine transformation:

$$L_\zeta:\quad\frac{(\xi\sqrt{\lambda^2+S}\cos\beta-\lambda\eta\sin\beta)^2}{a^2}+\frac{(\xi\sqrt{\lambda^2+S}\sin\beta+\lambda\eta\cos\beta)^2}{b^2}=1. \qquad (60)$$

We term the ellipse L_ζ as "induced". In order to construct the conformal map of the exterior of the ellipse L_ζ on the exterior of the circle $|\,\zeta\,|=1$ one should know the lengths A and B of the ellipse semi-axes (60) and the angle ψ between these semi-axes and the axes $O\xi\eta$ in the plane $\zeta=\xi+i\eta$ of the ellipse disposition (60). Let us turn the plane ζ at the angle $(-\psi)$, where the angle ψ is selected in such a way as to have, after going from the variables ξ, η to the variables ξ_1, η_1 in (60) in accordance with the axis-turning formulae, the coefficient equal to zero at the product $\xi_1\eta_1$ of the coordinates in τ_1-plane (upon completion of the turn):

$$\mathrm{tg}\,2\psi=\frac{2\lambda\sqrt{\lambda^2+S}(a^2-b^2)\sin2\beta}{S(a^2+b^2)(a^2-b^2)(2\lambda^2+S)\cos2\beta},$$

$$\frac{1}{A^2}=\frac{(\sqrt{\lambda^2+S}\cos\beta\cos\psi-\lambda\sin\beta\sin\psi)^2}{a^2}+\frac{(\sqrt{\lambda^2+S}\sin\beta\cos\psi+\lambda\cos\beta\sin\psi)^2}{b^2}, \qquad (61)$$

$$\frac{1}{B^2}=\frac{(\sqrt{\lambda^2+S}\cos\beta\sin\psi+\lambda\sin\beta\cos\psi)^2}{a^2}+\frac{(\sqrt{\lambda^2+S}\sin\beta\sin\psi-\lambda\cos\beta\cos\psi)^2}{b^2}.$$

The angle ψ is chosen so that the semi-axes A and B of the induced ellipse at $S=0$ could change into the semi-axes a and b of the ellipse (59) respectively. This condition provides for $\psi=-\beta$ at $S=0$. In τ-plane the ellipse equation (60) is canonical. The map of the exterior of this ellipse on the exterior of the circle, provided $k=d\zeta/d\tau\Big|_\infty>0$ [2]:

$$\zeta=k\left(\tau+\frac{C}{\tau}\right),\qquad k=\frac{A+B}{2},$$
$$C=\frac{A-B}{A+B}e^{2i\psi}=qe^{2i\psi},\qquad q=\frac{A-B}{A+B}. \qquad (62)$$

10. Impact of force on ellipse

We shall use the values (62) in (33), (44) for definition of the turbulent part of the hydroelectromagnetic impact of the flow on the ellipse. The turbulent part of the moment in this task is according to reference [2] $\mathbf{M}_1 = 0$. Zero too is the inertial part of the hydroelectromagnetic force: $\mathbf{R}_2 = 0$. This is established according to the results of reference [2] involving the map (62) in compliance with the theorem on residues. Therefore the index "unity" in (33) is dropped. To simplify the force (33) we shall use (61), (62) and (63). Then

$$\begin{aligned}\frac{X}{\rho V^2 a'} &= \frac{1}{2}\pi\lambda^2\sqrt{\lambda^2+S}\Big\{\lambda\Big[(A+B)^2 \\ &-(A-B)^2\cos 2\psi\Big]\cos\alpha - \sqrt{\lambda^2+S}(A^2-B^2) \\ &\times\sin 2\psi\sin\alpha\Big\} - \pi ab\lambda^2\cos\alpha, \\ \frac{Y}{\rho V^2 a'} &= \frac{1}{2}\pi(\lambda^2+S)\Big\{\sqrt{\lambda^2+S}\Big[(A+B)^2 \\ &+(A-B)^2\cos 2\psi\Big]\sin\alpha \\ &-\lambda(A^2-B^2)\sin 2\psi\cos\alpha\Big\} - \pi ab\lambda^2\sin\alpha.\end{aligned} \tag{63}$$

In (63) X and Y are projections of the force on the axis that are associated with the magnetic field ($\mathbf{x}^0 = \mathbf{B}^0$). The projections X', Y' of this force on the ellipse axis (59), the ram P and the lift N are associated with X and Y via the formulae

$$X'+iY' = (X+iY)e^{i\beta}, \quad P+iN = (X+iY)e^{-i\alpha} \tag{64}$$

At $S = 0$ the formulae (63) as a particular case, include the impact force of the flow on the ellipse in the absence of magnetic field. Let us find this force. At $S = 0$ from the formulae (61) we find that $\psi = -\beta$, $A = a/\lambda$, $B = b/\lambda$; the relations (63) and (64) coming out with ($a = 1$, $b = b'/a'$, $\lambda^2 = \kappa a'/V$, κ – the turbulent resistance coefficient [2])

$$\begin{aligned}X+iY &= \frac{1}{2}\pi\rho V\kappa[(a'^2+b'^2)e^{i\alpha} \\ &-(a'^2-b'^2)e^{-i(\alpha-2\beta)}], \\ X'+iY' &= \pi\rho\kappa[b'^2\cos(\alpha+\beta) \\ &+ia'^2\sin(\alpha+\beta)], \\ P+iN &= \frac{1}{2}\pi\rho V\kappa[a'^2 \\ &+b'^2(a'^2-b'^2)e^{-2i(\alpha+\beta)}]\end{aligned} \tag{65}$$

$\alpha+\beta$ – the angle of incidence, and, despite the Euler-d'Alambert paradox,

$$\begin{aligned}P &= \pi\rho V\kappa\left[a'^2\sin^2(\alpha+\beta) + b'^2\cos(\alpha+\beta)\right] \\ N &= \frac{1}{2}\pi\rho V\kappa\left(a'^2-b'^2\right)\sin 2(\alpha+\beta).\end{aligned}$$

From the formulae 65 at $b' = 0$, $\_ = 0$ we obtain the impact of force of the non-electricity conductive fluid on the plate which is $2a$ wide at the angle of incidence α (the dimensionality primes are omitted):

$$X = 0, \qquad Y = \pi\rho\kappa a^2\sin\alpha. \tag{66}$$

In the ideal fluid flow the force acts on the plate at the right angle relative to its plane.

Let us investigate the general expression (64) for the lift N and the resistance P for any values of the angles α and β. We shall re-write the formulae (63):

$$\begin{aligned}\frac{2X}{\pi\rho V\kappa a'^2} &= \sqrt{1+t^2}\Bigg\{\Bigg[\lambda^2(A^2+B^2) \\ &+2\left(\lambda^2 AB - \frac{ab}{\sqrt{1+t^2}}\right)\Bigg]\cos\alpha - \lambda^2(A^2-B^2) \\ &\times(\cos\alpha\cos 2\psi + \sqrt{1+t^2}\sin\alpha\sin 2\psi)\Bigg\}, \\ \frac{2Y}{\pi\rho V\kappa a'^2} &= (1+t^2)\sqrt{1+t^2}\Bigg\{\Bigg[\lambda^2(A^2+B^2) \\ &+2\left(\lambda^2 AB\frac{ab}{\sqrt{1+t^2}}\right)\Bigg]\sin\alpha - \lambda^2(A^2-B^2) \\ &\times(\sqrt{1+t^2}\sin\alpha\cos 2\psi - \cos\alpha\sin 2\psi)\Bigg\}.\end{aligned} \tag{67}$$

Introduced here is a magnetohydrodynamic parameter (see. [2])

$$t = \sqrt{\frac{S}{\lambda^2}} = \sqrt{\frac{\sigma B_*^2}{\rho\kappa}} = \frac{1}{\tau}. \tag{68}$$

We shall transform the formulae (61), by getting rid in the expressions for A and B of the angle ψ. an ambiguity appearing while expressing $\sin 2\psi$ in terms of $\operatorname{tg} 2\psi$. It is eliminated by using the relations $B = 0$ if $b = 0$ (the proof is omitted). Then by using (61) and (68)

$$\frac{\lambda^4 A^2 B^2}{a^2 b^2} = \frac{1}{1+t^2}, \quad \frac{a^2 b^2}{\lambda AB} = ab\sqrt{1+t^2}. \tag{69}$$

Next, we use the following formulae (64)–(68):

$$\begin{aligned}\frac{4P}{\pi\rho\kappa V a'^2\sqrt{1+t^2}} &= \lambda^2[(2+t^2)(A^2+B^2) \\ &+t^2(A^2-B^2)\cos 2\psi] - \lambda^2[t^2(A^2+B^2) \\ &+(2+t^2)(A^2-B^2)\cos 2\psi]\cos 2\alpha \\ &-2\lambda^2(A^2-B^2)\sqrt{1+t^2}\sin 2\psi\sin 2\alpha, \\ \frac{4N}{\pi\rho\kappa V a'^2\sqrt{1+t^2}} &= \lambda^2[t^2(A^2+B^2)+ \\ &(2+t^2)(A^2-B^2)\cos 2\psi]\sin 2\alpha \\ &-2\lambda^2(A^2-B^2)\sqrt{1+t^2}\sin 2\psi\cos 2\alpha.\end{aligned} \tag{70}$$

We shall now use the formulae (69) and omit the derivations again:

$$P = \pi\rho\kappa V a'^2 \left[a'^2 \sin^2(\alpha+\beta) + b'^2 \cos^2(\alpha+\beta)\right]\sqrt{1+\frac{S}{\lambda^2}},$$
$$N = \frac{1}{2}\pi\rho\kappa V(a'^2 - b'^2)\sqrt{1+\frac{S}{\lambda^2}}\sin 2(\alpha+\beta).$$

According to the formulae (64) and (70), it follows that $X' + iY' = (P + iN)\exp[i(\alpha+\beta)]$,

$$X' = \pi\rho\kappa V b'^2\sqrt{1+\frac{S}{\lambda^2}}\cos(\alpha+\beta),$$
$$Y' = \pi\rho\kappa V a'^2\sqrt{1+\frac{S}{\lambda^2}}\sin(\alpha+\beta).$$

Consequently, the magnetic field does not exert influence on the force direction, influencing only its value by increasing it by $\sqrt{1+\sigma B_*^2/(\rho\kappa)}$ (the scale B_* is determined in reference [2]) times relative to the value of the self-same force in the turbulent flow in the absence of the magnetic field. The force is independent of the applied magnetic field direction vs. ellipse.

11. Effect of moment

We shall now define the hydrodynamic moment [2], which would collapse the elliptical cylinder in the turbulent MHD flow:

$$\mathbf{M}_2 = \frac{\mathbf{M}_2'}{\rho V^2 L_*^2} = \frac{2\pi k^2}{\lambda^2(\lambda^2+S)}\mathrm{Im}\,[\overline{W}(C\overline{W} - W)] - S\frac{\lambda^2(J_\xi - J_\eta) + SJ_\xi}{\lambda^4(\lambda^2+S)^2} \quad (71)$$
$$J_\xi = \int_0^{2\pi}(\mathrm{Im}\,F)^2\xi\mathrm{d}\,\xi, \quad J_\eta = \int_0^{2\pi}(\mathrm{Re}\,F)^2\eta\mathrm{d}\,\eta,$$
$$F = i\frac{\overline{W} - We^{-2i\theta}}{1 - Ce^{-2i\theta}}.$$

where the integration is made over the azimuth of θ polar coordinates in τ-plane. The differentials $\xi\mathrm{d}\,\xi$ and $\eta d\eta$ on the contour $\tau = e^{i\theta}$ considering (62)

$$\xi\mathrm{d}\,\xi = -\frac{1}{2}k^2\mathrm{Im}\,\left[(1+\overline{C})^2 e^{2i\theta}\right]\mathrm{d}\,\theta,$$
$$\eta\mathrm{d}\,\eta = \frac{1}{2}k^2\mathrm{Im}\,\left[(1-\overline{C})^2 e^{2i\theta}\right]\mathrm{d}\,\theta.$$

We shall introduce the derived differentials into the integrals (71):

$$J_\xi = \frac{1}{8}k^2\mathrm{Im}\,\left[(1+\overline{C})^2 G(-1)\right],$$
$$J_\eta = \frac{1}{8}k^2\mathrm{Im}\,\left[(1-\overline{C})^2 G(1)\right].$$

$G(\nu) = \int_0^{2\pi}(F + \nu\overline{F})^2 e^{2i\theta}\mathrm{d}\,\theta$. We introduce the function F from (71) into the integral $G(\nu)$ and integrate over the contour $\tau = e^{i\theta}$, while using the theorem on residues of the function $F(\tau)$ relative to the two-fold poles $\tau_{1,2} = \pm\sqrt{C}$ inside the circumference $|\tau| = 1$. Then the moment (71) would be:

$$\mathbf{M}_2 = \frac{\mathbf{M}_2'}{\rho V^2 L_*^2} = \frac{\pi k^2}{4\lambda^4(\lambda^2+S)^2} \times \mathrm{Im}\left\{-\left[S(1+\overline{C}) + 2\lambda^2(1+\overline{C}^2)\right]\frac{Sw^2}{1-q^2} + \left[8\lambda^2(\lambda^2+S) + 4\lambda^2 S\overline{C} + S^2(1+\overline{C})^2\right]w\overline{W}\right\}.$$

The numbers k,C and q are determined in (62). The numbers w and W done in (33). The derived expression does not contain any singularity at $q = 1$ (flow round plate):

$$\mathrm{Im}\,\left\{\left[S(1+\overline{C})^2 + 2\lambda^2(1+\overline{C}^2)\right]w^2\right\} = -(1-q^2)\mathrm{Im}\left\{\left[(S+2\lambda^2)(1+q^2) + 2SC\right]\overline{W}^2 + 2(S+2\lambda^2)C|W|^2\right\} \Rightarrow$$
$$\mathbf{M}_2 = \frac{\pi k^2}{4\lambda^4(\lambda^2+S)^2}\mathrm{Im}\left\{CS\left[CS + 4(S+2\lambda^2)\right]|W|^2 + \left[(S+2\lambda^2)^2 C + S(S+2\lambda^2)(1+3q^2) + S^2(C + q^2\overline{C})\right]\overline{W}^2\right\}.$$

Next, we employ (33) and (62) ($C = q(\cos 2\psi + i\sin 2\psi)$):

$$\frac{\mathbf{M}'}{\rho V^2 a'^2} = -\frac{\pi(A+B)^2}{16\lambda\sqrt{\lambda^2+S}}\Big\{S^2(1+q)^3 + 2\lambda^2 S(1+q)(1+3q) + 8q\lambda^4 - 2q\left[2(2\lambda^2+S)^2 + S^2(1+q^2)\right] \times \sin^2\psi\Big\}\sin 2\alpha + \frac{\pi(A^2-B^2)}{16\lambda^2(\lambda^2+S)}\Big\{8\lambda^2(\lambda^2+S)^2 - (1-q^2)\lambda^2 S^2 + \left[2qS^3 + (2\lambda^2+S)^2(1+q^2)S^2 - 8\lambda^2(2\lambda^2+S)(\lambda^2+S)\right]\sin^2\alpha - 4qS^2(\lambda^2 + S\sin^2\alpha)\sin^2\psi\Big\}\sin 2\psi,$$
$$q = \frac{A-B}{A+B}. \quad (72)$$

The subscript of the moment $\mathbf{M}_2'$ is dropped, because the turbulent part of the moment $\mathbf{M}_1' = 0$. From the formulae (61) and (72) it follows that the moment $\mathbf{M}'$

depends on the ratio b'/a' of the lengths of the ellipse semi-axes, on the angles β between the induction vector $\mathbf{B}^0$ of the applied magnetic field and larger part of the ellipse α and between the vectors $(V)^0$ of the running stream velocity and $\mathbf{B}^0$, on the relation (τ-plane is needed no more) $\tau = \sqrt{\lambda^2/S} = \sqrt{\rho\kappa/(\sigma B_*^2)}$ of the roots of the turbulent resistance dimensionless coefficient vs. the Stewart number [2], and also on the square of the running stream velocity and fluid flow density:

$$\mathbf{M}' = \rho V^2 a'^2 \mathbf{f}\left(\alpha, \beta, \frac{b'}{a'}, \tau\right). \tag{73}$$

12. Particular cases

We shall now get ourselves convinced that there is no hydroelectromagnetic moment during the flow-around: the equality $\mathbf{M}' = 0$ for the circle from (72) is not straightforward. By assuming that in the formulae (61) $a = b$, we obtain $\psi = 0$, $A = (\lambda^2 + S)^{-1/2}$, $B = \lambda^{-1}$. Then $q = \dfrac{\tau - \sqrt{\tau^2+1}}{\tau + \sqrt{\tau^2+1}} = -1 + 2\tau(\sqrt{\tau^2+1} - \tau)$. That is why the value, which is independent of ψ in (72), and which can be only different from zero $a = b$, is proportional

$$\begin{aligned}(1+q)^3 &+ 2\tau^2(1+q)(1+3q) + 8\tau^4 q \\ &= 8\tau^3(\sqrt{\tau^2+1} - \tau)^3 + 8\tau^4(\sqrt{\tau^2+1} - \tau)^2 \\ &\quad \times 8\tau^3\sqrt{\tau^2+1}(\sqrt{\tau^2+1} - \tau)^2 \\ &= 8\tau^3(\sqrt{\tau^2+1} - \tau)^2(\sqrt{\tau^2+1} - \tau + \tau - \sqrt{\tau^2+1}) = 0.\end{aligned}$$

In the absence of the field (at $S = 0$) the formula (72) must change into the expression [4] for the moment acting on the ellipse on part of the ideal fluid. In this event, from (61)it follows that $\psi = -\beta$, $Aa = a/\lambda$, $B = b/\lambda$, and the formula (72) comes out with ($q = (a-b)/(a+b)$)

$$\begin{aligned}\frac{\mathbf{M}'}{\rho V^2 a'^2} = &-\frac{\pi(a+b)^2}{16\lambda^4}(8q\lambda^4 - 16q\lambda^4\sin^2\beta)\sin 2\alpha \\ &+ \frac{\pi(a^2-b^2)}{16\lambda^6}(8\lambda^6 16\lambda^6\sin^2\beta)\sin 2\beta.\end{aligned}$$

Whence, as it should be, $\mathbf{M}' = -\dfrac{1}{2}\pi\rho V^2(a'^2 - b'^2)\sin 2(\alpha+\beta)$ Interesting are the cases of the co-linearity ($\alpha = 0$, $\alpha = \pi$) and orthogonality ($\alpha = \pi/2$) of the vectors $\mathbf{B}^0$ and $\mathbf{V}^0$. If $\alpha = \pi/2$ ($B^0 \perp V^0$), then from (72)

$$\begin{aligned}M' = &\frac{\pi\rho V^2 a'^2(A^2 - B^2)}{16\lambda^2} \\ &\times [S^2(1 + 2q\cos 2\psi + q^2) - 8\lambda^4]\sin 2\psi. \end{aligned}\tag{74}$$

If the vectors B^0 and V^0 are co-linear ($\alpha = 0$ or $\alpha = \pi$), then

$$\begin{aligned}M' = -&\frac{\pi\rho V^2 a'^2(A^2 - B^2)}{16(\lambda^2 + S)}[S^2(1 - 2q\cos 2\psi + q^2) \\ &- 8(\lambda^2 + S)^2]\sin 2\psi.\end{aligned}\tag{75}$$

13. Moment-free flow-around

From the formulae (61) for the angle ψ it follows that the moments (74) and (75) are zero at $\beta = 0$ and $\beta = \pi/2$ when the applied magnetic field is directed along one of the axes of the ellipse. Then the running flow is also directed along one of the axes of the ellipse, i.e. the flow-around is symmetrical. Is there a possibility of such choice of the applied magnetic field induction that the moment (72) relative to the centre-of-gravity of the ellipse could be zero during the non-symmetrical flow-around? The answer in the positive to this question is of applied interest, since there appears the feasibility of decreasing the moment load to the bodies round which the flow passes owing to an external factor of non-hydrodynamic nature. The formal putting of this question looks like as follows: Has the equation $M' = \rho V^2 a'^2 f(\alpha, \beta, b'/a', \tau) = 0$, where the function f is determined in (73) and (72), a solution relative to τ at fixed values of α, β and b'/a'. In the absence of magnetic field, the moment of the non-symmetrical flow passing round the ellipse is not zero at all times. Let us reply to the question addressed in the particular case, where the ellipse degenerates into plate ($b' = 0$), and the running flow is perpendicular to the applied magnetic field ($\alpha = \pi/2$), i.e. where the moment is determined according to (73) (there is another particular case given below). At $\beta \neq 0$, $\beta \neq \pi/2$ the induced ellipse $\xi^2/A^2 + \eta^2/B^2 = 1$ in magnetic field is never a circumference ($A \neq B$, the proof is omitted). Therefore at $\beta \neq 0$, $\beta \neq \pi/2$ the equation $M' = 0$ at $\alpha = \pi/2$ is equivalent to the equation

$$q^2 + 2q\cos 2\psi + 1 - 8\tau^4 = 0 \tag{76}$$

In the general case ($b \neq 0$), where this equation connects the parameter τ with the ratio of the lengths of the axes b of the ellipse round which the flow passes to the angle β between the running flow velocity and applied magnetic field induction, the connection (76) is intricate enough (see. (61)) and should be studied using the numeric methods. At $b = 0$, where the flow is running up against the plate, the equation (76) is drastically reduced, since the plate $b = 0$ is the induced ellipse for the plate $B = 0$. As noted above, the angle ψ should be selected out of the condition $\psi = -\beta$ at $S = 0$ ($\tau = \infty$). Consequently

$$\operatorname{tg}\psi = -\frac{\sqrt{\tau^2+1}}{\tau}\operatorname{tg}\beta, \quad b = 0. \tag{77}$$

From the formula (77) we find

$$\cos\psi = \frac{\tau\cos\beta}{\sqrt{\tau^2+\sin^2\beta}}, \quad b=0. \tag{78}$$

The proof dropped, we note that the induced ellipse for the plate $b=0$ is a plate with the half-length

$$A = \frac{1}{\lambda}\sqrt{\frac{\tau^2+\sin^2\beta}{\tau^2+1}}, \quad (B=0,\ \ b=0). \tag{79}$$

Let us get back to the equation (76), which for the plate $b=0$ considerin (79) ($q=1$) has the form $\cos^2\psi = 2\tau^4$ or (see. (78)), $\cos^2\beta = \dfrac{2\tau^2(\tau^2+1)}{2\tau^2+1}$. This is achieved only at $2\tau^4 \le 1$ i.e. at $\sqrt{2}\rho\kappa \le \sigma B_*$. Thus, if the applied field induction vector forms with the plate the angle

$$\beta = \arccos\left(\tau\sqrt{\frac{2(\tau^2+1)}{2\tau^2+1}}\right).$$

If the running flow velocity is orthogonal to the induction vector, then at the condition $\sqrt{2}\rho\kappa \le \sigma B_*$ for the hydrodynamic and electromagnetic characteristics of the flow, there is no moment that would collapse the plate in the flow relative to its centre-of-gravity. If $\mathbf{B}^0 \perp \mathbf{V}^0$, then at $\beta \ne 0$, $\beta \ne \pi/2$ the moment is always not zero, since in this case according to (74) the equation $M' = 0$ is reduced to the one $\sin^2\psi = 2(\tau^2+1)$. Let now $\beta = 0$ ($\psi = 0$) or $\beta = \pi/2$ ($\psi = -\pi/2$) – the magnetic induction vector is directed along one of the axes of the ellipse that has the flow passing round it. At $\beta = 0$ the magnetic field is directed along the larger axis of the ellipse (59) and the formula (72) has now

$$q = \frac{a\lambda - b\sqrt{\lambda^2+S}}{a\lambda + b\sqrt{\lambda^2+S}}, \qquad \beta = 0: \ \frac{M'}{\rho V^2 a'^2} = (\pi(a\lambda + b\sqrt{\lambda^2+S})^2[S^2(1+q)^3 + 2\lambda^2 S(1+q)(1+3q) + 8q\lambda^4]) /(16\lambda^3(\lambda^2+S)^{3/2})\sin 2\alpha \tag{80}$$

If plate ($b=0$, $a=1$) has the flow passing round it and applied magnetic field is directed along the plate, then

$$M' = -\frac{1}{2}\pi\rho V^2 a'^2\sqrt{1+\frac{S}{\lambda^2}}\sin 2\alpha, \quad \beta = 0. \tag{81}$$

At $S=0$ hence the classical result is produced, since in this case α is the angle of incidence. At $\beta = \pi/2$, $\psi = -\pi/2$ the field is directed along the smaller axis of the ellipse (59), and

$$q = \frac{a\sqrt{\lambda^2+S} - b\lambda}{b\lambda + a\sqrt{\lambda^2+S}}, \quad \beta = \frac{\pi}{2}: \ \frac{M'}{\rho V^2 a'^2} = -\pi(b\lambda + a\sqrt{\lambda^2+S})^2[S^2(1-q)^3 + 2\lambda^2 S(1-q)(1-3q) - 8q\lambda^4] /(16\lambda^3(\lambda^2+S)^{3/2})\sin 2\alpha. \tag{82}$$

When plate has the flow passing round it (the magnetic field is directed normal to it)

$$M' = -\frac{\pi\rho V^2 a'^2}{2\sqrt{1+S/\lambda^2}}\sin 2\alpha.$$

Here the angle of incidence $\alpha_1 = \alpha + \pi/2$, whence $2\alpha = 2\alpha_1 - \pi$, so that the formula at $S=0$ becomes classical. It is not difficult to show that with the exception of the symmetric cases $\alpha = 0$ or a $\alpha = \pi/2$ and at $a = b$ – the circular flow-around – the moments (80) and (82) do not go to zero at no matter whatever the ratio of the ellipse semi-axes to the physical parameter τ.

14. Effect of moment on plate

If there is the plate ($b=0$, $B=0$, $q=1$), in the flow, then the formula (72) for the moment can be expressed in terms of the initial parameters, angles α, β and parameters λ and S [2]. We now include the formulae (72) in (77)–(79).

$$\sin\psi = \operatorname{tg}\psi\cos\psi = -\sqrt{\frac{\lambda^2+S}{\lambda^2+S\sin^2\beta}}\sin\beta,$$

$$A^2\sin 2\psi = -\frac{\sin 2\beta}{\lambda\sqrt{\lambda^2+S}}.$$

Then the moment that would collapse the plate in the flow is

$$M' = -\frac{\pi\rho V^2 a'^2}{4\lambda\sqrt{\lambda^2+S}}\Big\{[S + (2\lambda^2+S)\cos 2\beta]\sin 2\alpha + (S(2\lambda^2+S)(1 - cos2\alpha\cos 2\beta) + 4\lambda^2(\lambda^2+S)\cos 2\alpha) /(2(\lambda^2+S\sin^2\beta))\sin 2\beta\Big\}. \tag{83}$$

At $\beta = 0$, $\beta = \pi/2$ we have now again (80) and (82). At $\alpha = 0$ or $\alpha = \pi$

$$M' = -\frac{\pi[2\lambda^2(\lambda^2+S) + S(2\lambda^2+S)\cos^2\beta]\sin 2\beta}{4\lambda\sqrt{\lambda^2+S\sin^2\beta}} \times \rho V^2 a'^2. \tag{84}$$

At $\alpha = \pi/2$

$$M' = \frac{\pi[2\lambda^2(\lambda^2+S) - S(2\lambda^2+S)\cos^2\beta]\sin 2\beta}{4\lambda\sqrt{\lambda^2+S\sin^2\beta}} \times \rho V^2 a'^2. \tag{85}$$

The formulae (84) and (85), are easy to verify as well as the particular cases of the formulae (74) and (75) respectively. Evidently, in the absence of the applied field symmetry relative to the plate ($\beta \ne 0$,

$\beta \neq \pi/2$) the moment (84) does not go to zero, while the moment (85) is zero at $\alpha = 0$ and at

$$\beta = \arccos\left(\lambda\sqrt{\frac{2(\lambda^2+S)}{2\lambda^2+S}}\right) = \arccos\left(\tau\sqrt{\frac{2(\tau^2+1)}{2\tau^2+1}}\right).$$

This value of the angle β has already been obtained above. The formula (83) enables to analyze the cases of the zero moment that would collapse the plate in the flow in the absence of the flow symmetry ($\beta \neq 0$, $\beta \neq \pi/2$, $\alpha \neq 0$, $\alpha \neq \pi/2$). So, $M' = 0$ at $\cos 2\beta = \cos 2\alpha = -S(2\lambda^2+S)^{-1}$, i.e. at

$$\alpha = \beta = \pm\frac{1}{2}\left(\pi - \arccos\frac{S}{2\lambda^2+S}\right) + k\pi, \quad k = 0, \pm 1, \pm 2.$$

Here the case $\alpha = -\beta$ is trivial, since the running flow is co-linear to the plate. Yet, at $\alpha = \beta$ the angle of incidence is 2β and determined by the electromagnetic and hydrodynamic parameters of the flow. If $\lambda^2 \gg S$, then the formula (83) changes into the classical result for the moment that would collapse the plate in the flow in the absence of the magnetic field. At $S \gg \lambda$ (in strong magnetic fields)

$$M' \sim -\frac{\pi\rho V^2 a'^2 S\sqrt{S}(1-\cos 2\alpha\cos 2\beta)\sin 2\beta}{8\lambda(\lambda^2+S\sin^2\beta)}.$$

In the denominator in the parenthesis the small value of λ^2 is shown for the case $\beta \sim 0$.

In applied field the moment (72) can differ from zero at the symmetrical flow-around, where the flow velocity at infinity is directed along one of the axes of the ellipse, if, in the meantime, the external magnetic induction vector is not directed along the same or some other axis ($\beta \neq 0$, $\beta \neq \pi/2$). We shall limit ourselves to the case of the flow passing round plate. Let the running flow be orthogonal to the plate. Then $\alpha = -\beta + \pi/2$, $\sin 2\alpha = \sin 2\beta$, $\cos 2\alpha = -\cos 2\beta$, and during the transverse flow round the plate:

$$M' = -\frac{\pi\rho V^2 a'^2 S(2\lambda^2+S)\sin 2\beta}{4\lambda\sqrt{\lambda^2+S}(\lambda^2+S\sin^2\beta)} \qquad (86)$$

If the vectors $\mathbf{V}^0$ and $\mathbf{B}^0$ form an acute angle α (in the transverse field), the moment acts so as to turn the vector $\mathbf{V}^0$ in the direction of the vector $\mathbf{B}^0$ following the least arc. In the counter-streaming magnetic field, when the angle α is obtuse, the moment $\mathbf{M}'$ rotates the plate from the vector $\mathbf{B}^0$ to the vector $\mathbf{V}^0$ following, once again, the least arc. If in (86) one can change to the angle α between the vectors $\mathbf{B}^0$ and $\mathbf{V}^0$, then all of the cases get embraced with one formula:

$$\mathbf{M}' = \frac{\pi\rho V^2 a'^2 S(2\lambda^2+S)\cos 2\alpha}{4\lambda\sqrt{\lambda^2+S}(\lambda^2+S\cos^2\alpha)}[\mathbf{V}^0 \times \mathbf{B}^0].$$

In the case of the longitudinal flow passing round plate ($\alpha = -\beta$) the hydroelectromagnetic moment is zero in any direction of applied magnetic field.

15. Reduction of formulae

Using the formulae (69) we shall derive the expression (72) for the moment in terms of the initial problem parameters: the semi-axes a, b of the ellipse (59), the angles α, β and a magnetohydrodynamic parameter t (see. (68)). We shall do so for the cases of orthogonality and co-linearity of the vectors $\mathbf{V}$ and $\mathbf{B}$.

If $\alpha = \pi/2$, then by employing (74), (62) – for q – and (68), we shall obtain as follows

$$\frac{M'}{\rho V^2 a'^2} = \frac{\pi\lambda^2(A^2-B^2)\sin 2\psi}{8(A+B)^2} \times \left\{\left[A^2+B^2+(A^2-B^2)\cos 2\psi\right]t^4 - 4(A+B)^2\right\}$$

Let us multiply and divide the right-hand part of the last expression for the relation $(a^2b^2/(\lambda^2A^2B^2))^2$, go from the values A^2, B^2 and AB respectively, to $a^2b^2/(\lambda^2B^2)$, $a^2b^2/(\lambda^2A^2)$, and $a^2b^2/(\lambda^2AB)$ and use the formulae (69). Then ($\alpha = \pi/2$)

$$\frac{M'}{\rho V^2 a'^2} = \big(\pi(a^2-b^2)\big[(t^4-2t^2-4)(a^2+b^2) - 8ab\sqrt{1+t^2} + t^2(2+t^2)(a^2-b^2)\cos 2\beta\big]\sin 2\beta\big) \Big/ \big(8\big[a^2+2ab\sqrt{1+t^2}+b^2 + t^2(a^2\sin^2\beta+b^2\cos^2\beta)\big]\sqrt{1+t^2}\big). \quad (87)$$

Similarly, if $\mathbf{V} \parallel \mathbf{B}$ ($\alpha = 0$ or $\alpha = \pi$), then from (75)

$$\frac{M'}{\rho V^2 a'^2 \sin 2\beta} = \big(\pi(a^2-b^2)\big[2(1+t^2)(a^2+2ab\sqrt{1+t^2}+b^2) + t^2(2+t^2)(a^2-b^2)(a^2\sin^2\beta+b^2\cos^2\beta)\big]\big) \Big/ \big(8\big[a^2+2ab\sqrt{1+t^2}+b^2 + t^2(a^2\sin^2\beta+b^2\cos^2\beta)\sqrt{1+t^2}\big]\big]. \quad (88)$$

The formulae (87) and (88) change, at $b = 0$, into the appropriate expressions (85) and (84) for the hydroelectromagnetic moment action on the plate. During the non-symmetric flow ($\beta \neq \pi$, $\beta \neq \pi/2$) round the ellipse the expression (87) is not zero. If the running flow and applied magnetic field are co-linear, then the moment acting on the ellipse in the flow does not go to zero no matter whatever the relation between the geometric parameter $b = b'/a'$ ($a = 1$) of the ellipse and the magnetohydrodynamic parameter t^2. Regarding the plate, this property has been established above. If $\mathbf{B} \perp \mathbf{V}$, then the moment

(87) goes to zero in that event, where the magnetic induction vector $\mathbf{B}$ forms with the larger axis of the ellipse the angle

$$\beta = \frac{1}{2}\arccos\big((4+2t^2-t^4)(a^2+b^2)+8ab\sqrt{1+t^2}\big) \Big/ \big(t^2(2+t^2)(a^2-b^2)\big) \quad (89)$$

provided that

$$\left|\frac{(4+2t^2-t^4)(a^2+b^2)+8ab\sqrt{1+t^2}}{t^2(2+t^2)(a^2b^2)}\right| \le 1$$

or $(a=1)$ at (see. (68))

$$b^2+\sqrt{2}+b\sqrt{b^2+2(\sqrt{2}+1)} \le \frac{\sigma B_*^2}{\rho\kappa} \le 1+\frac{1+\sqrt{1+2b^2(\sqrt{2+1})}}{b^2}.$$

At $b=0$ this expression (as well as the angle (89)), changes into the constraints $(\sqrt{2}\rho\kappa \le \sigma B_*^2)$ and the angle for the case of flow passing round plate.

16. Conclusions

This research solved a number of problems defining the hydroelectromagnetic effects of EMF flow of the ideal fluid in applied magnetic field. Resolved, as well, are the problems of hydrodynamic effects of fluid flow outside EMF and *without employing the notion of "circulation"*. Integral representations are constructed such that characterize the moment effects of fluid on the contour. By using the results of this work [3], via the smaller parameter method, the solution of the addressed problems can be used to determine the effects on the contour in EMF at the large Reynolds numbers. The Authors hope that the results of this research attract specialists in magnetic hydrodynamics, hydrodynamics of flows passing round contours at the large Reynolds numbers and in aerodynamics.

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