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The Electron Mass from Deformed Special Relativity

Contents

Abstract

Deformed Special Relativity (DSR) is a generalization of Special Relativity based on a deformed Minkowski space, i.e. a four-dimensional space-time with metric coefficients depending on the energy. We show that, in the DSR framework, it is possible to derive the value of the electron mass from the space-time geometry via the experimental knowledge of the parameter of local Lorentz invariance breakdown, and of the minkowskian threshold energy $E_{0,em}$ for the electromagnetic interaction.

1. Introduction

In the last years, two of the present authors (F.C. and R.M.) proposed a generalization of SR based on a "deformation" of space-time, assumed to be endowed with a metric whose coefficients depend on the energy of the process considered[(1)]. Such a formalism (*Deformed Special Relativity*, DSR) applies in principle to *all* four interactions (electromagnetic, weak, strong and gravitational) — at least as far as their nonlocal behavior and nonpotential part is concerned — and provides a metric representation of them (at least for the process and in the energy range considered)[(1−4)]. Moreover, it was shown that such a formalism is actually a five-dimensional one, in the sense that the deformed Minkowski space is embedded in a larger Riemannian manifold, with energy as fifth dimension[(5)].

In this paper, we will show that the DSR formalism yields an expression of the electron mass m_e in terms of the parameter δ of local Lorentz invariance (LLI) breakdown and of the threshold energy for the gravitational metric, $E_{0,grav}$ (i.e. the energy value under which the metric becomes Minkowskian). This allows us to evaluate m_e from the (experimental) knowledge of such parameters.

The organization of the paper is as follows. In Sec. 2 we briefly introduce the concept of deformed Minkowski space, and give the explicit forms of the phenomenological energy-dependent metrics for the four fundamental interactions. The LLI breaking parameter δ_{int} for a given interaction is introduced in Sec.3. In Sec. 4 we assume the existence of a stable fundamental particle interacting gravitationally, electromagnetically and weakly, and show (by imposing some physical requirements) that its mass value (expressed in terms of $\delta_{e.m.}$ and $E_{0,grav}$) is just the electron mass. Sec. 5 concludes the paper.

2. Deformed Special Relativity in Four Dimensions (DSR4)

2..1 Deformed Minkowski Space-Time

The generalized ("deformed") Minkowski space $\widetilde{M}_4$ (DMS4) is defined as a space with the same local coordinates x of M_4 (the four-vectors of the usual Minkowski space), but with metric given by the metric tensor[1]

$$\eta_{\mu\nu}(E) = diag\left(b_0^2(E), -b_1^2(E), -b_2^2(E), -b_3^2(E)\right) \stackrel{\text{ESC off}}{=} \delta_{\mu\nu}\left[\delta_{\mu 0} b_0^2(E) - \delta_{\mu 1} b_1^2(E) - \delta_{\mu 2} b_2^2(E) - \delta_{\mu 3} b_3^2(E)\right], \quad (1)$$

($\forall E \in R_0^+$) where the $\left\{b_\mu^2(E)\right\}$ are dimensionless, real, positive functions of the energy[1]. The generalized interval in $\widetilde{M}_4$ is therefore given by ($x^\mu = (x^0, x^1, x^2, x^3) = (ct, x, y, z)$, with c being the usual light speed in vacuum) (ESC on)

$$ds^2 = b_0^2(E) c^2 dt^2 - (b_1^2(E) dx^2 + b_2^2(E) dy^2 + b_3^2(E) dz^2) = \eta_{\mu\nu}(E) dx^\mu dx^\nu = dx * dx. \quad (2)$$

The last step in (2) defines the scalar product $*$ in the deformed Minkowski space $\widetilde{M}_4$[2]. It follows immediately that it can be regarded as a particular case of a Riemann space with null curvature.

Let us stress that, in this formalism, the energy E is to be understood as the energy of a physical process measured by the detectors via their electromagnetic interaction in the usual Minkowski space. Moreover, E is to be considered as a dynamical variable (on the same footing as the space-time coordinates), because it specifies the dynamical behavior of the process under consideration, and, via the metric coefficients, it provides us with a dynamical map — in the energy range of interest — of the interaction ruling the given process. Let's recall that the use of momentum components as dynamical variables on the same foot of the space-time ones can be traced back to Ingraham[8]. Dirac[9], Hoyle and Narlikar[10] and Canuto et al.[11] treated mass as a dynamical variable in the context of scale-invariant theories of gravity.

Moreover — as already stressed in the Introduction — the 4-d deformed Minkowski space can be naturally embedded in a 5-d Riemann space, with energy as fifth metrical coordinate[5]. Curved 5-d spaces have been considered by several Authors[12]. On this respect, the DSR formalism is a kind of generalized (non-compactified) Kaluza-Klein theory, and resembles, in some aspects, the so-called "Space-Time-Mass" (STM) theory (in which the fifth dimension is the rest mass), proposed by Wesson[13] and studied in detail by a number of authors[14].

2..2 Energy-Dependent Phenomenological Metrics for the Four Interactions

As far as the phenomenology is concerned, we recall that a local breakdown of Lorentz invariance may be envisaged for all four fundamental interactions (electromagnetic, weak, strong and gravitational) whereby *one gets evidence for a departure of the space-time metric from the Minkowskian one* (at least in the energy range examined). The experimental data analyzed were those of the following four physical processes: the lifetime of the (weakly decaying) K_s^0 meson[15]; the Bose-Einstein correlation in (strong) pion production[16]; the superluminal photon tunneling[17]; the comparison of clock rates in the gravitational field of Earth[18]. A detailed derivation and discussion of the energy-dependent phenomenological metrics for all the four interactions can be found in refs. [1-4]. Here, we confine ourselves to recall their following basic features:

1) Both the electromagnetic and the weak metric show the same functional behavior, namely

$$\eta(E) = diag(1, -b^2(E), -b^2(E), -b^2(E)), \quad (3)$$

$$b^2(E) = \begin{cases} (E/E_0)^{1/3}, & 0 < E \leq E_0 \\ 1, & E_0 < E \end{cases} = 1 + \theta(E_0 - E)\left[\left(\frac{E}{E_0}\right)^{1/3} - 1\right], E > 0 \quad (4)$$

(where $\theta(x)$ is the Heaviside theta function) with the only difference between them being the threshold energy E_0, i.e. the energy value at which the metric parameters are constant, i.e. the metric becomes Minkowskian ($\eta_{\mu\nu}(E \geq E_0) \equiv g_{\mu\nu} = diag(1, -1, -1, -1)$); the fits to the experimental data

[1] In the following, we shall employ the notation "*ESCon*" ("*ESCoff*") to mean that the Einstein sum convention on repeated indices is (is not) used.

[2] Notice that our formalism — in spite of the use of the word "deformation" — has nothing to do with the "deformation" of the Poincaré algebra introduced in the framework of quantum group theory (in particular the so-called κ-deformations)[6]. In fact, the quantum group deformation is essentially a modification of the commutation relations of the Poincaré generators, whereas in the DSR framework the deformation concerns the metrical structure of the space-time (although the Poincaré algebra is affected, too[7]).

yield

$$E_{0,e.m,} = (4.5 \pm 0.2)\,\mu eV, \quad E_{0,weak} = (80.4 \pm 0.2)\,GeV. \tag{5}$$

Notice that for either interaction the metric is isochronous, spatially isotropic and *"sub-Minkowskian"*, i.e. it approaches the Minkowskian limit from below (for $E < E_0$). Both metrics are therefore Minkowskian for $E > E_{0w} \simeq 80$ GeV, and then our formalism is fully consistent with electroweak unification, which occurs at an energy scale ~ 100 GeV.

Let us recall that the phenomenological electromagnetic metric (3)–(5) was derived by analyzing the propagation of evanescent waves in undersized waveguides(16). It allows one to account for the observed superluminal group speed in terms of a nonlocal behavior of the waveguide, just described by an effective deformation of space-time in its reduced part(3). As to the weak metric, it was obtained by fitting the data on the meanlife of the meson K_s^0 (experimentally known in a wide energy range ($30 \div 350$ GeV)(14)), thus accounting for its apparent departure from a purely Lorentzian behavior(1,19).

2) For the strong interaction, the metric was derived(2) by analyzing the phenomenon of Bose-Einstein (BE) correlation for π-mesons produced in high-energy hadronic collisions(16). Such an approach permits to describe the BE effect as the decay of a "fireball" whose lifetime and space sizes are directly related to the metric coefficients $\{b^2_{\mu,strong}(E)\}$, and to avoid the introduction of "ad hoc" parameters in the pion correlation function(2). The strong metric reads

$$\eta_{strong}(E) = diag(b^2_{0,strong}(E), - b^2_{1,strong}(E), -b^2_{2,strong}(E), -b^2_{3,strong}(E)), \tag{6}$$

$$b^2_{1,strong}(E) = \left(\frac{\sqrt{2}}{5}\right)^2, \quad b^2_{2,strong}(E) = \left(\frac{2}{5}\right)^2, \forall E > 0, \tag{7}$$

$$b^2_{0,strong}(E) = b^2_{3,strong}(E) = \begin{cases} 1, & 0 < E \leq E_{0,strong} \\ (E/E_{0,strong})^2, & E_{0,strong} < E \end{cases} = 1 + \theta(E - E_{0,strong})\left[\left(\frac{E}{E_{0,strong}}\right)^2 - 1\right], \quad E > 0 \tag{8}$$

with

$$E_{0,strong} = (367.5 \pm 0.4)\ \text{GeV}. \tag{9}$$

Let us stress that, in this case, contrarily to the electromagnetic and the weak ones, *a deformation of the time coordinate occurs*; moreover, *the three-space is anisotropic*, with two spatial parameters constant (but different in value) and the third one variable with energy like the time one.

3) The gravitational energy-dependent metric was obtained(4) by fitting the experimental data on the relative rates of clocks in the Earth gravitational field(18). Its explicit form is[3]:

$$\eta_{grav}(E) = diag(b^2_{0,grav}(E), - b^2_{1,grav}(E), -b^2_{2,grav}(E), -b^2_{3,grav}(E)); \tag{10}$$

$$b^2_{0,grav}(E) = b^2_{3,grav}(E) = \begin{cases} 1, & 0 < E \leq E_{0,grav} \\ \frac{1}{4}(1 + E/E_{0,grav})^2, & E_{0,grav} < E \end{cases} = 1 + \theta(E - E_{0,grav.})\left[\frac{1}{4}\left(1 + \frac{E}{E_{0,grav.}}\right)^2 - 1\right], \quad E > 0 \tag{11}$$

with

$$E_{0,grav} = (20.2 \pm 0.1)\,\mu eV. \tag{12}$$

Intriguingly enough, this is approximately of the same order of magnitude of the thermal energy corresponding to the 2.7^oK cosmic background radiation in the Universe[4].

Notice that the strong and the gravitational metrics are *over-Minkowskian* (namely, they approach the Minkowskian limit from above ($E_0 < E$), at least for their coefficients $b^2_0(E) = b^2_3(E)$).

3. LLI Breaking Factor and Relativistic Energy in DSR

The breakdown of standard local Lorentz invariance (LLI) is expressed by the LLI breaking factor parameter δ (19). We recall that two different kinds of LLI violation parameters exist: the isotropic (essentially obtained by means of experiments based on the propagation of e.m. waves, e.g. of the Michelson-Morley type), and the anisotropic ones (obtained by experiments of the Hughes-Drever type(19), which test the isotropy of the nuclear levels).

[3] The coefficients $b^2_{1,grav}(E)$ and $b^2_{2,grav}(E)$ are presently undetermined at phenomenological level.

[4] It is worth stressing that the energy-dependent gravitational metric (10)–(12) is to be regarded as a *local* representation of gravitation, because the experiments considered took place in a neighborhood of Earth, and therefore at a small scale with respect to the usual ranges of gravity (although a large one with respect to the human scale).

In the former case, the LLI violation parameter reads[19]

$$\delta = \left(\frac{u}{c}\right)^2 - 1, \quad u = c + v \tag{13}$$

where c is, as usual, the speed of light in vacuo, v is the LLI breakdown speed (e.g. the speed of the preferred frame) and u is the new speed of light (i.e. the *"maximal causal speed"* in Deformed Special Relativity[1]). In the anisotropic case, there are different contributions δ^A to the anisotropy parameter from the different interactions. In the HD experiment, it is A=S, HF, ES, W, meaning strong, hyperfine, electrostatic and weak, respectively. These correspond to four parameters δ^S (due to the strong interaction), δ^{ES} (related to the nuclear electrostatic energy), δ^{HF} (coming from the hyperfine interaction between the nuclear spins and the applied external magnetic field) and δ^W (the weak interaction contribution).

In our framework, we can define δ as follows:

$$\delta_{int.} \equiv \frac{m_{in.,int.} - m_{in.,grav.}}{m_{in.,int.}} = 1 - \frac{m_{in.,grav.}}{m_{in.,int.}} \tag{14}$$

where $m_{in.,int.}$ is the inertial mass of the particle considered with respect to the given interaction [5]. In other words, we assume that the *local* deformation of space-time corresponding to the interaction considered, and described by the metric (1), gives rise to a *local violation* of the Principle of Equivalence for interactions different from the gravitational one. Such a departure, just expressed by the parameter δ_{int}, does constitute also a measure of the amount of LLI breakdown. In the framework of DSR, δ_{int} embodies the geometrical contribution to the inertial mass, thus discriminating between two different metric structures of space-time.

Of course, if the interaction considered is the gravitational one, the Principle of Equivalence strictly holds, i.e.

$$m_{in.,grav.} = m_g \tag{15}$$

where m_g is the gravitational mass of the physical object considered, i.e. it is its "gravitational charge" (namely its coupling constant to the gravitational field).

Then, we can rewrite (1) as:

$$\delta_{int.} \equiv \frac{m_{in.,int.} - m_g}{m_{in.,int.}} = 1 - \frac{m_g}{m_{in.,int.}} \tag{16}$$

and therefore, when the particle is subjected only to gravitational interaction, it is

$$\delta_{grav.} = 0 \tag{17}$$

[5] Throughout the present work, "int." denotes a physically detectable fundamental interaction, which can be operationally defined by means of a phenomenological energy-dependent metric of deformed minkowskian type.

In DSR the relativistic energy, for a particle subjected to a given interaction and moving along $\widehat{x^i}$, has the form[1]:

$$E_{int} = m_{in.,int} u^2_{i,int}(E) \widetilde{\gamma}_{int}(E) = m_{in.,int} c^2 \frac{b^2_{0,int}(E)}{b^2_{i,int}(E)} \times \left[1 - \left(\frac{v_i b_{i,int}(E)}{c b_{0,int}(E)}\right)^2\right]^{-1/2} \tag{18}$$

where $\underline{u}_{int}(E)$ is the maximal causal velocity for the interaction considered (i.e. the analogous of the light speed in SR), given by[1,21]

$$\underline{u}_{int}(E) \equiv \left(c\frac{b_{0,int}(E)}{b_{1,int}(E)}, c\frac{b_{0,int}(E)}{b_{2,int.}(E)}, c\frac{b_{0,int}(E)}{b_{3,int}(E)}\right) \tag{19}$$

In the non-relativistic (NR) limit of DSR, i.e. at energies such that

$$v_i \ll u_{i,int}(E) \tag{20}$$

eq.(17) yields the following NR expression of the energy corresponding to the given interaction:

$$E_{int,NR} = m_{in,int} u^2_{i,int}(E) = m_{in.,int} c^2 \frac{b^2_{0,int}(E)}{b^2_{i,int}(E)} \tag{21}$$

In the case of the gravitational metric (10)–(12), we have

$$\frac{b^2_{0,grav}(E)}{b^2_{3,grav}(E)} = 1, \forall E \in R_0^+ \tag{22}$$

Therefore, for $i = 3$, eqs.(17) and (20) become, respectively ($v_3 = v$):

$$E_{grav} = m_g c^2 \left[1 - \left(\frac{v}{c}\right)^2\right]^{-1/2} = m_g c^2 \gamma, \tag{23}$$

$$E_{grav,NR} = m_g c^2 \tag{24}$$

namely, the gravitational energy takes its *standard, special-relativistic values.*

This means that the special characterization (corresponding to the choice $i = 3$) of eqs.(17) and (20) within the framework of DSR relates the gravitational interaction with SR, which is — as well known – based on the electromagnetic interaction in its minkowskian form.

4. The Electron as a Fundamental Particle and Its "Geometrical" Mass

Let us consider for E the threshold energy of the gravitational interaction:

$$E = E_{0,grav} \tag{25}$$

where $E_{0,grav.}$ is the limit value under which the metric $\eta_{\mu\nu,grav}(E)$ becomes minkowskian (at least in its known components). Indeed, from eqs. (10),(13) it follows:

$$\eta_{\mu\nu,grav}(E) = diag(1, -b_{1,grav}^2(E), -b_{2,grav}^2(E), -1) \stackrel{\text{ESC off}}{=} \delta_{\mu\nu}\left[\delta_{\mu 0} - \delta_{\mu 1} b_{1,grav}^2(E) - \delta_{\mu 2} b_{2,grav}^2(E) - \delta_{\mu 3}\right], \quad \forall E \in (0, E_{0,grav}]$$

Notice that at the energy $E = E_{0,grav}$ the electromagnetic metric (3),(4) is minkowskian, too (because $E_{0,grav} > E_{0,em}$).

On the basis of the previous considerations, it seems reasonable to assume that the physical object (particle) p with a rest energy (i.e. gravitational mass) just equal to the threshold energy $E_{0,grav}$, namely

$$E_{0,grav.} = m_{g,p}c^2, \tag{26}$$

must play a fundamental role for either e.m. and gravitational interaction. We can e.g. hypothesize that p corresponds to the lightest mass eigenstate which experiences both force fields (i.e. , from a quantum viewpoint, coupling to the respective interaction carriers, the photon and the graviton). As a consequence, p must be intrinsically stable, due to the impossibility of its decay in lighter mass eigenstates, even in the case such a particle is subject to weak interaction, too (i.e. it couples to all gauge bosons of the Glashow-Weinberg-Salam group $SU(2) \times U(1)$, not only to its electromagnetic charge sector).

Since, as we have seen, for $E = E_{0,grav}$ the electromagnetic metric is minkowskian, too, it is natural to assume, for p:

$$m_{in,p,e.m.} = m_{in,p} \tag{27}$$

namely its inertial mass is that measured with respect to the electromagnetic metric.

Then, due to the Equivalence Principle (see eq. (2)), the mass of p is characterized by

$$p : \begin{cases} m_{in,p,grav} = m_{g,p} \\ m_{in,p,e.m.} = m_{in,p} \end{cases} \tag{28}$$

Therefore, for such a fundamental particle the SSLI breaking factor (14) of the e.m. interaction becomes:

$$\delta_{e.m.} = \frac{m_{in,p} - m_{g,p}}{m_{in,p}} = 1 - \frac{m_{g,p}}{m_{in,p}} \Leftrightarrow m_{g,p} = m_{in,p}\left(1 - \delta_{e.m.}\right) \tag{29}$$

Replacing (26) in (29) yields:

$$E_{0,grav} = m_{in,p}\left(1 - \delta_{e.m.}\right)c^2 \Leftrightarrow m_{in,p} = \frac{E_{0,grav}}{c^2}\frac{1}{1 - \delta_{e.m.}} \tag{30}$$

Eq.(30) allows us to evaluate the inertial mass of p from the knowledge of the electromagnetic LLI breaking parameter $\delta_{e.m.}$ and of the threshold energy $E_{0,grav}$ of the gravitational metric.

The lowest limit to the LLI breaking factor of electromagnetic interaction has been recently determined by an experiment based on the detection of a DC voltage across a conductor induced by the steady magnetic field of a coil(22). The value found in [22] corresponds to

$$1 - \delta_{e.m.} \cong 4 \cdot 10^{-11} \tag{31}$$

Then, inserting the value (12) for $E_{0,grav}$ [6] and (31) in (30), we get

$$m_{in,p} = \frac{E_{0,grav}}{c^2}\frac{1}{1 - \delta_{e.m.}} \geq \frac{2 \cdot 10^{-5}}{4 \cdot 10^{-11}}\frac{eV}{c^2} = 0.5\frac{MeV}{c^2} = m_{in,e} \tag{32}$$

(with $m_{in,e}$ being the electron mass) where the $\geq$ is due to the fact that in general the LLI breaking factor constitutes an *upper limit* (i.e. it sets the scale *under which* a violation of LLI is expected). If experiment [22] *does indeed provide evidence* for a LLI breakdown (as it seems the case, although further confirmation is needed), eq.(33) yields $m_{in,p} = m_{in,e}$. We find therefore the amazing result that *the fundamental particle p is nothing but the electron* e^- *(or its antiparticle* e^+ [7] *).* The electron is indeed the lightest massive lepton (pointlike, non-composite particle) with electric charge, and therefore subjected to gravitational, electromagnetic and weak interactions, but unable to weakly decay due to its small mass. Consequently, e^- (e^+) shares all the properties we required for the particle p, whereby it plays a fundamental role for gravitational and electromagnetic interactions.

5. Conclusions

The formalism of DSR describes – -among the others —, in geometrical terms (via the energy-dependent deformation of the Minkowski metric) the breakdown of Lorentz invariance at local level (parametrized by the LLI breaking factor δ_{int}). We have shown that within DSR it is possible - on the basis of simple and plausible assumptions - to evaluate the inertial mass of the electron e^- (and therefore

[6] Let us recall that the value of $E_{0,grav}$ was determined by fitting the experimental data on the slowing down of clocks in the Earth gravitational field(18). See ref.[4].

[7] Of course, this last statement does strictly holds only if the CPT theorem mantains its validity in the DSR framework, too. Although this problem has not yet been addressed in general on a formal basis, we can state that it holds true in the case we considered, since we assumed that the energy value is $E = E_{0,grav}$, corresponding to the minkowskian form of both electromagnetic and gravitational metric.

of its antiparticle, the positron e^+) by exploiting the expression of the relativistic energy in the deformed Minkowski space $\widetilde{M}_4(E)_{E\in R_0^+}$, the explicit form of the phenomenological metric describing the gravitational interaction (in particular its threshold energy), and the LLI breaking parameter for the electromagnetic interaction $\delta_{e.m.}$.

Therefore, the inertial properties of one of the fundamental constituents of matter and of Universe do find a "geometrical" interpretation in the context of DSR, by admitting for local violations of standard Lorentz invariance.

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