PACS №: 01.55.+b, 31.15.Ew, 71.10.-w, 71.15.Mb

Jaime Keller
CMS, Institute of General Physics,
Technical University of Vienna
Getreidemarkt 9/134,
A-1060 Vienna, Austria and
Department of Physics and Theoretical Chemistry,
Facultad de Química and Facultad de Estudios Superiores Cuautitlán,
Universidad Nacional Autónoma de México 04510
México D.F., apartado 70-528, MÉXICO
e-mail: keller@servidor.unam.mx
e-mail: keller@cms.tuwien.ac.at

A Theory of the Neutrino from START

Contents

Abstract

A comprehensive theory of the neutrino is derived from a basic approach to the theoretical structures of physics we have called START. All the observed properties of the neutrino are contained in the wave equation defining the field.

1. Introduction

In [1–4] we have presented a derivation of the basic structures of physics from first principles, within a general framework we have called START. The theory is an independent and fundamental formulation for the description of an extended system in space.

The methodology deduces the theory from the consideration of a distribution of action $\mathfrak{w}(X) \geq 0$ (in units of action density in a four dimensional manifold X called space–time, this density is equivalent to an energy density over space or to a pressure in the Tolman's sense). In START we have, besides space–time, an additional one dimensional manifold w for a total of five dimensions. To analyze the distribution $\mathfrak{w}(X)$ we introduced the concept of carriers densities ρ_c for a set $\{c\}$ of carriers of energy–momentum (a density at rest for an observer appears for another observer in a motion relative to the first as a current). We have given in [5] a formal definition of the carrier density which represents an elementary type of physical object.

In these series of papers we have developed a comprehensive theory of matter which has followed Scheme 1.1

In this program the principles refer to the existence of an unified geometry for space, time and action intervals. A fundamental principle is also the requirement of the state of the system to correspond to that with the least observer's space–time trajectories. The postulates include the geometrical union of the manifold of those variables through the systematic use of the concept of carriers and the requirement that exchanges of action should occur in Planck's quanta.

1.1. Energy, Momentum and Gauge Fields

Because our basic quantity is a distribution $\mathfrak{w}(X)$ in space–time X, it is natural to introduce the gradients of this quantity, defined to correspond to the ordinary concepts of energy $\mathfrak{E}$ (time derivative of action) and of momentum $\mathfrak{P}$ (space derivatives of action).

There are in the theory two different contribution to energy and momentum. First the quantities defined in the paragraph above and, second, the quantities which will be called relative energy or relative momentum. This second contribution arising from the choice made to describe the physical phenomena as interactions between bodies. These bodies in our approach are called "carriers".

A main principle, after the introduction of the space–time–action 5-D manifold, is the Principle of Relativity, which in our formulation is stated as follows

Principle of Space–Time–Action Relativity. In a space–time–action manifold an unstructured observer can not determine its own state of motion, he can only determine the relative motion of other bodies in relation to itself and among the other bodies themselves.

According to this principle an observer will describe the bodies of the physical nature in a motion relative to the observer itself and accordingly will assign to those bodies a momentum which we define as *motion originated momentum* (usually denoted by p), and from the relative motion among the bodies themselves assign a momentum which we define as *interaction originated momentum* (usually denoted by $q_r A_{or}$ where the interaction 'charge" q_b is then explicitly introduced both for the action receiving body r as well as for the action originating body o). The interaction originated momentum is the result of a non-unique description procedure, this freedom of definition will mathematically appear as a "gauge freedom" in the formulations below.

1.2. Mass, Charges, Action, Space and Time

In our formalism **"action"**, as a fundamental variable, is distributed among a set of **carrier (of action) fields**. An action density $\mathfrak{w}(\mathbf{x}, t)$ is the fundamental concept defining all three: space (parametrized by the vector $\mathbf{x}$), time (parametrized by the scalar t, $X = \{\mathbf{x}, t\}$) and action density (parametrized by the scalar valued analytical function $\mathfrak{w}(\mathbf{x}, t)$) as **primitive** concepts from which all other physical quantities will be derived or at least related directly or indirectly. The different forms of distributing the action among this carriers define the carriers themselves. Here action (action density) is given the status of an independent variable (function).

1.3. Observer's Coordinates Frame Embedded in a Reference Geometrical Frame

We will use as a background manifold a 5-D quadratic space, we will call ZRW frame. This background manifold will be a flat, pseudo-euclidean quadratic space with diagonal metric $g_{AB} = diag\{1, -1, -1, -1, -1\}$ with $ds^2 = g_{\mu\nu} dy^\mu dy^\mu$. The reference Z corresponds to a 1-D quadratic space with metric $g_{00} = 1$, the reference R corresponds to a 3-D quadratic space $dl^2 = g_{ij} dy^i dy^j$ with metric $g_{ij} = diag\{-1, -1, -1\}; i, j = 1, 2, 3$ and the reference W corresponds to a 1-D quadratic space with metric $g_{44} = -1$. As a guide to the reader we have in mind an asimtotic representation of time, space and action (which in German carry the name Zeit, Raum and Wirkung), as far as in our theory the basic quantities are made to correspond to what an observer measures as space–time intervals and computes as action increments. The concept "observer" corresponds to a composite primitive notion of a scientist studying the physical nature and measuring quantities which in general are sets of two real numbers whose difference corresponds to a physical quantity, the five physical quantities dy^μ the observer measures are: an oriented distance in space dy^i, a time difference $dy^0 \doteq cdt$ and an action increment $dy^4 \doteq \kappa_0 da$.

Таблица 1..1:

SPACE + TIME + ACTION (5-D manifold)
+
FUNDAMENTAL PRINCIPLES
and
POSTULATES
⇓
START GEOMETRY and Carriers
⇓
Density Functional Theory
↙ ↓ ↘
General Relativity   Quantum Mechanics   Standard Model

A mapping will be made in such a form that for another observer the measured quantities dx^μ will be $dx^\mu = B^\mu_\nu(\mathbf{x})dy^\nu$, that is this second observer is measuring local weighted quantities dx^μ which are local projections of dS. Our theory will allow the selection of this projection $B^\mu_\nu(\mathbf{x})$.

A principle in the theory is that the interval dS^2 should have an extreme value, which will translate (and enlarge) in our formalism the Maupertois least action principle, transforming it into a geometric condition. The time-like intervals will be expressed as equivalent distances using a universal velocity (distance to time interval ratio): the velocity of light c. The action-like increments will also be expressed as equivalent distance using a universal constant κ_0 (distance to action ratio, in our formalism the ratio of the electron's Compton wave length r_0 to Planck's constant h, corresponding to the inverse of the product of the electron mass and the velocity of light $\kappa_0 = r_0/h = 1/m_0c$, this defines the electron as the reference carrier). In practice the use in this geometry of the geometrical least interval principle, a simple approach is to consider the existence of an evolution parameter τ and then write $dx^0 = cv^0\alpha_0 d\tau$, $dx^i = v^i\alpha_0 d\tau$ with $dx^4\kappa_0 da$, $da = \sum\limits_\mu p_\mu dx^\mu = \sum\limits_\mu p_\mu v^\mu \alpha_0 d\tau$, to obtain all quantities as functions of the evolution parameter τ. The least interval principle will then read

$$\delta \int dS = 0, \tag{1}$$

and will be equivalent from variational calculus to the application of the operator

$$\frac{\partial}{\partial x^\mu} - \frac{d}{d\tau}\frac{\partial}{\partial \overset{\bullet}{x}{}^\mu} \tag{2}$$

to the interval and equating the result to zero. Here the symbol $\overset{\bullet}{x}{}^\mu = \partial x^\mu/\partial\tau$, v^μ is a space–time velocity $\sum\limits_\mu v^\mu v_\mu = 1$ and α_0 is the ratio of $v^0 d\tau$ to a corresponding time interval.

In this form a geometric formulation of the description of space–time and action for each observer is obtained and the transformation between observers will be a geometrical transformation. As any observer will be accepted as equivalent, our theory is also a mathematical formulation of the principle of relativity in the Poincaré–Einstein sense.

1.4. Carriers and Physical Bodies

Within our fundamental formulation we will have to define properties of the fields we call "carriers".

One of the basic concepts we are using is that of a carrier, in principle a carrier of action, defined in such a form that it is also a carrier of energy–momentum, angular momentum, charges and other physical properties. A special type of carrier corresponds to what is usually related to the word "matter". The experimental setup which refers to matter is always related to an interaction. An interaction is an exchange or sharing of energy–momentum, the actual values linearized by considering them proportional to a characteristic (of that portion of matter) called "charge". Otherwise the charges are to be defined in our theory from a geometrical analysis.

We have already defined as a primary concept a quantity "density of action" $\mathfrak{w}(X)$ at every point of the space–time frame of reference, this action is distributed among a set of carriers c in such a form that $\mathfrak{w}(X) = \sum\limits_c \mathfrak{w}_c(X)$. The space–time derivatives of action are called energy–momentum and correspond to a distribution among carriers in such a form that for each carrier for which a charge e is defined the attributed density of energy–momentum per unit charge is

$$\begin{aligned} \mathfrak{E}_e(X) &= \frac{\partial a_e(X)}{\partial t}, \\ \mathbf{p}_e &= \left(\frac{\partial a_e(X)}{\partial x^i} + \Delta_R\, p_{e,i}\right) e^i \end{aligned} \tag{3}$$

in the second term and additional quantity $\Delta_R\, p_{e,i}e^i$

has been introduced indicating that momentum has acquired a double significance: that amount which is related to the rate of change of action with respect to position and that amount which is shared, adding to zero, among the carriers. In the case of energy it is not customary to add a relative energy to each of the carriers with signatures which allow to cancel the added amounts of energy, this is originated from the observational fact that we can not define one body moving in the forward direction of time and another in the backward direction, while this is not a restriction with respect to moving in the forward or backward direction of the space coordinates.

The rates of change of relative energy and momentum are called forces and are then given by

$$\begin{aligned} \mathbf{F} &= \nabla \mathfrak{E}_e(X) + \frac{\partial \mathbf{p}_e}{\partial t}, \\ \mathbf{B} &= \left(\frac{\partial p_{e,i}(X)}{\partial x^j} \right) e^j \times e^i. \end{aligned} \tag{4}$$

These forces have, themselves, a space–time dependance, then the third derivatives (which correspond to second derivatives of energy–momentum) are related by the Leibnitz derivation rules

$$\begin{aligned} \nabla \times \mathbf{F} &= -\frac{\partial}{\partial t}\mathbf{B}, \\ \nabla \cdot \mathbf{F} &= \frac{1}{\epsilon_0}\rho \text{ (definition)}, \\ \nabla \cdot \mathbf{B} &= \frac{1}{\mu_0}\frac{\partial}{\partial t}\mathbf{F}, \\ \nabla \cdot \mathbf{B} &= 0 \end{aligned} \tag{5}$$

here, for the case of $\nabla \cdot \mathbf{F} = \frac{1}{\epsilon_0}\rho$, there are no second or third derivative to which $\nabla \cdot \mathbf{F}$ can be equated and then this scalar (one component of a vector quantity $j = \rho e^0 + j_i e^i$) is called the charge density (time-like component of the current j). This quantity is a definition of the current and in particular of the charge density. Whenever the forces have a divergency different from zero we say that there is a charge (current), this is a geometrical definition. The quantities $\mathbf{F}$ and $\mathbf{B}$ mix with each other under space–time (Lorentz) transformations. A carrier for which a current of charges can be defined is by definition a body. A body corresponds to our hitherto undefined concept of matter.

Our study below will show that we can not define an elementary body if other properties, besides a charge, are not given to the carrier.

In our presentation the word "particle" is systematically avoided as for many authors it refers to a "point" body, with no spatial dimensions. The word body, otherwise, conveys the idea of some space distribution, a fundamental concept in any theory where the existence of a minimum amount of action, Planck's action h, is considered fundamental. Point-like distributions can only be introduced as a practical tool for handling a distribution confined to a region of space small in relation to the total system's volume.

1.5. Formal Definition of Carrier Fields

Within our fundamental formulation we will have to define properties of the fields we call "carriers". We follow our presentation in [5].

A carrier-domain $\mathcal{B}$ is a connected open set whose elements can be put into bijective correspondence with the points of a region (domain in some instances) $\mathbf{B}$ of a Euclidean point space $\mathbb{E}$. $\mathbf{B}$ is referred to as a configuration of $\mathcal{B}$; the point in $\mathbf{B}$ to which a given element of $\mathcal{B}$ corresponds is said to be "*occupied*" by that *element*. If $\mathcal{X}$ denotes a representative *element* of $\mathcal{B}$ and $\mathbf{x}$ the position relative to an origin $\mathbf{0}$ of the point $\mathbf{x}$ occupied by $\mathcal{X}$ in $\mathbf{B}$, the preceding statement implies the existence of a function $\theta : \mathcal{B} \to \mathbf{B}_0$ ($\mathbf{B}_0$, stands for the totality of the positions relative to $\mathbf{0}$ of the points of $\mathbf{B}$) and its inverse $\Theta : \mathbf{B}_0 \longrightarrow \mathcal{B}$ such that

$$\mathbf{x} = \theta(\mathcal{X}), \qquad \mathcal{X} = \Theta(\mathbf{x}). \tag{6}$$

In a motion of a carrier-domain the configuration changes with time. Let t be a real variable, denoting time, and $I \in \mathbb{R}$ an interval (not necessarily bounded). If with each value of t in I, there is an associated unique configuration $\mathbf{B}_t$, of a carrier-domain $\mathcal{B}$, the family of configurations $\{\mathbf{B}_t : t \in I\}$ is called a motion of $\mathcal{B}$. This definition entails the existence of functions $\phi : \mathcal{B} \otimes I \to (\mathbf{B}_t)_0$ and $\Phi : \{(\mathbf{x}, t) : t \in I, \mathbf{x} \in (\mathbf{B}_t)_0\} \longrightarrow \mathcal{B}$ such that

$$\mathbf{x} = \phi(\mathcal{X}, t), \quad \mathcal{X} = \Phi(\mathbf{x}, t). \tag{7}$$

In a motion of $\mathcal{B}$ a typical element $\mathcal{X}$ occupies a succession of points which together form a curve in $\mathbb{E}$. This curve is called the path of $\mathcal{X}$ and is given parametrically by equation (7). The rate of change $\mathbf{v}$ of $\mathbf{x}$ in relation to t is called the velocity of the element $\mathcal{X}$, (our definitions run parallel to those of an extended body in continuum mechanics, see for example Spencer 1980 [13]). The velocity and the acceleration of $\mathcal{X}$ can be defined as the rates of change with time of position and velocity respectively as $\mathcal{X}$ traverses its path. "Kinematics" is this study of motion *per se*, regardless of the description in terms of physical forces causing it. The primitive concepts concerned are *position, time* and *carrier*, the latter abstracting into mathematical terms intuitive ideas about aggregations of matter capable of motion and deformation. The set of constitutive properties of the carriers, described below, will give useful, formal, existence to this concept. In space–time a body is a bundle of paths.

Equations (7) depict a motion of a carrier-domain as a sequence of correspondences between elements of $\mathcal{B}$ and points identified by their positions relative to a selected origin $\mathbf{0}$. At each $\mathcal{X}$ a scalar quantity is given, called carrier density $\varrho(\mathcal{X})$, such that if $\mathbf{x} = \phi(\mathcal{X}, t)$ then $\varrho(\mathcal{X}) \to \rho(\mathbf{x},t)$ defines a scalar field called local carrier density.

As mentioned a carrier will have physical significance through its set of properties. The value of the density of a carrier field can be defined through a set of fundamental scalar constants (main examples: mass and charges) such that the integral of the product of these constants and the density gives the experimentally attributed value of a property for that carrier. The density might become an indirect observable through the repeated measurement of those properties, but it is not an observable in itself. We will use an example. A carrier field (identified for example with an electron or a neutrino) will have a density $\rho(\mathbf{x}, t)$, such that if the property is Q we will obtain the definition $Q = \int\limits_{\bar{\mathrm{V}}} q(\mathbf{x}, t) d\mathbf{x} = \int\limits_{\bar{\mathrm{V}}} Q\rho(\mathbf{x}, t) d\mathbf{x}$ for all t in the system's volume $\bar{\mathrm{V}}$. This defines that Q as a constant property (in space and time) for that field (otherwise the variable quantity $q(\mathbf{x}, t)$ can be called "the density of Q "). The set of properties $\{Q\}$ characterizes a carrier field and in turn establishes the conditions for a density field to correspond to an acceptable carrier. This is for example the case of electromagnetism, and its daughter theory: elementary particles theory, where the set {mass, electric charge, weak charge, strong charge, spin} defines the 'elementary particle' one is dealing with.

1.6. Interaction Tensor

In $\mathcal{B}$ the carrier has as only properties its existence, whereas in $\mathbf{B}$ the carrier c has a distribution characterized by the density $\rho_c(\mathbf{x}, t)$. There is no restriction in defining a reference space $\mathbf{B}_R$ where the carrier exists in the points $\mathbf{x}$ with constant density $\rho^{(0)}$ occupying a volume V_0 such that $\rho^{(0)} V_0 = 1$. This two quantities are unobservable as far as any "observation" requires an "interaction", only then the distribution acquires a meaningful space dependence as a function, by definition, of an *external interaction* $\mathbf{V}(\mathbf{x}, t)$, which will be defined below. Here it is important to state that as a result of this interaction, and of the attributed properties of the carrier, the density evolves into a current: $\rho^{(0)} \Longrightarrow \overrightarrow{j_c^V}(\mathbf{x}, t)$. The first considerations are that $\rho^{(0)}$ is defined in a given proper frame of reference $\mathbb{E}^{(0)}$ with vectors $\{e_0^{(0)}, e_1^{(0)}, e_2^{(0)}, e_3^{(0)}\}$ and then a reference carrier's density current is defined in those points as $j^{(0)} = \rho^{(0)} e_0^{(0)}$. Second that, in the **observers** frame of reference $\mathbb{E}$ with vectors $\{e_0, e_1, e_2, e_3\}$, and as a result of the modification of the distribution caused by the interaction, the carrier's density current is given by

$$\overrightarrow{j_c^V}(\mathbf{x}, t) = \mathbf{R}_c^V(\mathbf{x}, t) \mathbf{S}_c^V(\mathbf{x}, t; \mathbf{x}') \mathbf{R}_c^{(0)} \rho^{(0)}(\mathbf{x}')\, e_0^{(0)},$$

with time-like component $\rho_c^V(\mathbf{x}, t) = \overrightarrow{j_c^V}(\mathbf{x}, t) \cdot e_0$.

That is the density is characterized by the properties of the carrier and the self-organization of the carrier which adapts to the external interactions.

Let $\mathbf{x}$ be the position of an arbitrary point $\mathcal{X}$ relative to an origin $\mathbf{o}$ and $\mathbf{S}(\mathbf{x}, t; \mathbf{x}')$ and $\mathbf{R}(\mathbf{x}, t)$ be respectively a positive definite symmetric tensor and a proper orthogonal tensor (fields) on $\mathbb{E}$.

We give first a geometrical interpretations of the actions of $\mathbf{S}$ and $\mathbf{R}$ on $\mathbf{x}$:

- $\mathbf{S}$ admits an spectral representation ($\mathbf{I} = \sum(\mathbf{p}_r \otimes \mathbf{p}_r)$)

$$\mathbf{S}(\mathbf{x}, t; \mathbf{x}') = \mathbf{S}(\mathbf{x}, t; \mathbf{x}') \sum(\mathbf{p}_r \otimes \mathbf{p}_r) = \sum \lambda_r(\mathbf{x}, t; \mathbf{x}')(\mathbf{p}_r \otimes \mathbf{p}_r)$$

  with λ_r and p_r the eigenvalues and eigenvectors of $\mathbf{S}$. The λ_r being positive and the associated proper vectors forming an orthonormal basis $\mathbf{p} = \{\mathbf{p}_r\}$ of $\mathbb{E}$. The tensor $\mathbf{S}(\mathbf{x}, t; \mathbf{x}')$ accordingly gives rise to a transformation in $\mathbb{E}$ consisting of proportional extensions, or stretches, of amounts $\lambda_r(\mathbf{x}, t; \mathbf{x}')$, in the mutually orthogonal directions defined by the unit proper vectors p_r. These directions are known as the principal axes of $\mathbf{S}$.

- The antisymmetric tensor $\mathbf{R}(\mathbf{x}, t)$ can be expressed in the form

$$\mathbf{R}(\mathbf{x}, t) = \mathbf{p} \otimes \mathbf{p} + (\mathbf{q} \otimes \mathbf{q} + \mathbf{r} \otimes \mathbf{r}) \cos\theta(\mathbf{x}, t) - (\mathbf{q} \otimes \mathbf{r} - \mathbf{r} \otimes \mathbf{q}) \sin\theta(\mathbf{x}, t)$$

  where $-\pi < \theta(\mathbf{x}, t) < \pi$ and $\{\mathbf{p}, \mathbf{q}, \mathbf{r}\}$ is an orthonormal basis in $\mathbb{E}$. The action of $\mathbf{R}(\mathbf{x}, t)$ on $\mathbf{x}$ may therefore be interpreted as a **rotation** in $\mathbb{E}$ by the amount $\theta(\mathbf{x}, t)$ about the axis through $\mathbf{o}$ in the direction defined by $\mathbf{p}$.

Second we give a *physical interpretation* to the transformations $\mathbf{S}(\mathbf{x}, t)$ and $\mathbf{R}(\mathbf{x}, t)$.

- The positive definite symmetric tensors $\mathbf{S}^V(\mathbf{x}, t; \mathbf{x}')$ describe the spatial distributions of the carriers density which should minimize the action of the system according to our physical principle that the state of the system should be that corresponding to a stationary action. Because the reference distribution is arbitrary we can replace the deformation $\mathbf{S}^V$ by an scale factor $S^V(\mathbf{x}, t)$ which describes the final carrier density distribution.

- The orthogonal tensor $\mathbf{R}^V(\mathbf{x}, t)$ describes the additional local rotation of the distribution arising from the effect of the external interaction.

- It was convenient to write the actual density distribution in the form

$$\rho^V(\mathbf{x},t) = \mathbf{R}^V(\mathbf{x},t)S^V(\mathbf{x},t)\mathbf{R}^{(0)}\rho^{(0)}$$

where the $\mathbf{R}^{(0)}$ describes the reference state of proper rotation of the carrier distribution. This reference rotation (a constant for an elementary carrier as defined in the next section) will be called *spin of the carrier* in accordance with the usual nomenclature. When the density $\rho(\mathbf{x},t)$ is described below this additional orthogonal tensor $\mathbf{R}^{(0)}$ will acquire its full physical significance. As a result of the definitions above we could in fact write

$$S^V(\mathbf{x},t) = S^V(\mathbf{V}(\mathbf{x},t)),\ \mathbf{R}^V(\mathbf{x},t) = \mathbf{R}^V(\mathbf{V}(\mathbf{x},t))$$

for the interaction tensors.

- The carriers density current can be written, if a multivector representation is given of $S^V(\mathbf{x},t)$ and $\mathbf{R}(\mathbf{x},t)$, as

$$\overrightarrow{j^V}(\mathbf{x},t) = S^V(\mathbf{x},t)\mathbf{R}^V(\mathbf{x},t)\mathbf{R}^{(0)}\rho^{(0)}e_0^{(0)} = \widehat{\mathbf{\Psi}}^V(\mathbf{x},t)\rho^{(0)}e_0^{(0)} = \Psi^V(\mathbf{x},t)\rho^{(0)}e_0^{(0)}\left(\Psi^V(\mathbf{x},t)\right)^\dagger$$

form which will acquire full meaning below. The carrier current amplitude function operators $\Psi^V(\mathbf{x},t)$ and $\left(\Psi^V(\mathbf{x},t)\right)^\dagger$ being either abstract multivectors or their matrix representations, describing the **extensions** and **Lorentz** (Poincaré) transformations defined by the operator $\mathbf{\Psi}^V$.

1.7. Composite, Decomposable, Elementary, Average and Average Description of Carriers

There are several forms of analyzing the density. Each one allows a physical interpretation. For example:

- A *composite* carrier is defined as one for which the density

$$\rho_C(\mathbf{x},t) = \sum_c A^c_C\rho_c(\mathbf{x},t) \tag{8}$$

with the definition of each of the $\rho_c(\mathbf{x},t)$ being also meaningful as a description of a carrier itself. In particular we can choose $\int_V \rho_c(\mathbf{x},t)d\mathbf{x} = \mathbf{1}$ and $A^c{=}N_c(t;C)$.

- Similarly a *non-decomposable* carrier is defined as one for which (8) applies but for which the meaning of each of the $\rho_c(\mathbf{x},t)$ can not be defined without reference to the global $\rho_C(\mathbf{x},t)$.

- An (*non-decomposable) elementary* carrier is one for which a single $\rho_c(\mathbf{x},t)$ is all it is needed; in this case we emphasize the discrete nature of an elementary carrier, but we do not assume a point–like or any internal structure for them.

- An *average* carrier is defined as one for which its density can be described as ($W = \sum_{c=1,n} A^c$)

$$\rho_A(\mathbf{x},t) = \frac{1}{W}\sum_{c=1,n} A^c_A\rho_c(\mathbf{x},t) \tag{9}$$

with the definition of each of the $\rho_c(\mathbf{x},t)$ being meaningful as a description of a carrier itself. Similarly an *average description of a carrier* can be defined either as a space average over carrier descriptions as in (9) or as a time average of a description, or sum of descriptions ($\overline{W} = \sum_{c=1,n} \frac{1}{\tau}\int_{t=t_0}^{t=t_0+\tau} A^c(t)dt$, the choice $\overline{W}=1$ presents less handling problems)

$$\overline{\rho(\mathbf{x})} = \frac{1}{\overline{W}}\sum_{c=1,n}\frac{1}{\tau}\int_{t=t_0}^{t=t_0+\tau} A^c(t)\rho_c(\mathbf{x},t)dt. \tag{10}$$

The introduction of restrictions (definitions) on the $A^c(t)\rho_c(\mathbf{x},t)$ terms defines the desired properties of the carriers. For example (with an Euler- Lagrange multiplier) a term of the form:

$A^c(t)\rho_c(\mathbf{x},t) = (B_{ij}(t))^c\left(\psi^i(\mathbf{x},t)\right)^\dagger\psi^j(\mathbf{x},t)$ can be used (see below) to introduce symmetry properties and interaction possibilities.

This paper is centered in the definition of the elementary carriers and their correspondence with the fields describing the elementary particles, in particular the neutrino and its partner particle the electron.

1.8. The Density

The conditions to be obeyed by the analytical function carrier density $\rho_c(\mathbf{x},t)$ are:

D1.– $\rho_c(\mathbf{x},t)$ is a real quantity $\rho_c(\mathbf{x},t)\subset\mathbb{R}$.

D2.– The density $0\le\rho_c(\mathbf{x},t)<\infty$ in order to represent a finite amount of charges and of action.

D3.– The derivatives of the density $-\infty<\partial_\mu\rho_c(\mathbf{x},t)<+\infty$ in order to represent a finite amount of energy–momentum.

If $\Psi(\mathbf{x},t)$ is an analytical quadratic integrable complex or multivector function, conditions D1, D2 and D3 are fulfilled identically if $\rho_c(\mathbf{x},t) = |\Psi_c(\mathbf{x},t)|^2$. Here $|f|^2$ means the real quadratic form of any more general function f, even if f itself is not necessarily a real

function and we define: if $|f|^2 = f^+f$ then $\partial_\mu |f|^2 = (\partial_\mu f^+)f + f^+(\partial_\mu f)$.

Condition D1 is fulfilled by the definition $\rho_c(\mathbf{x},t) = |\Psi_c(\mathbf{x},t)|^2$, D2 by the requirement of quadratic integrability, D3 by the definition $\partial_\mu |f|^2 = (\partial_\mu f^+)f + f^+(\partial_\mu f)$ and the analytical properties of $\Psi(\mathbf{x},t)$. It is seen that the conditions D1, D2, D3 and $\int_V \rho_c(\mathbf{x},t)d\mathbf{x} = N_c$ correspond to the $\Psi(\mathbf{x},t)$ being quadratic integrable Hilbert functions. In any Lagrangian type formulation this last definition of ρ can be used as a condition introduced via a Lagrange multiplier.

Gauge freedom Carrier density and density of action should be gauge invariant physical quantities, thus we need to develop a procedure which can allow gauge freedom, that is, a procedure which allows for arbitrary, but correct and useful, descriptions. This is fundamental to define the neutrino and the electron as interacting carriers.

Because the definition of carrier density depends on the attributed energy per carrier we can not separate the definition of the gauge, in a form compatible to the basic concept that the energy–momentum components are obtained by using the operator $i\hbar\partial_\mu$, from the definition of the carriers themselves.

The definition required by D1, D2 and D3 above $\rho_c = |\Psi_c|^2$, allows gauge independence. A set of Euler–Lagrange conditions and multipliers can be used to define the carriers and their desired properties. This procedure can be carried at any level of description, hence the universality of mathematical descriptions presented here, which in fact give a self existing status to density functional theory. There should be no confusion from the fact that $\rho_c = |\Psi_c|^2$, here proposed, is formally equivalent to the use of Wave Equations in Quantum Mechanics. This equivalence will be shown below to have far reaching consequences.

1.9. Schrödinger Amplitude Functions in START

The freedom of description of the energy-momentum partitioning is a fundamental issue in the construction of physical theories. Since at least the XIXth century an action function was introduced which was useful for this purpose, in the XXth century the de Broglie phase factor $\exp(i\triangle a/\hbar)$ allowed the freedom of energy-momentum description and the use of the gauge fields. Later the concept of non-commuting gauge fields was successfully introduced to describe a larger set of energy-momentum partitioning among carriers (fundamental interactions). We follow now the Schrödinger procedure.

1. Let the Schrödinger (1926) definition of action $\mathfrak{W}(\mathbf{x},t)$ in terms of an auxiliary function $\Psi(\mathbf{x},t)$ be
$$\mathfrak{W}(\mathbf{x},t) = K\ln\Psi(\mathbf{x},t) = -K\ln\Psi^\dagger(\mathbf{x},t),$$
that is: action is considered a sum of terms. The action $\mathfrak{W}(\mathbf{x},t)$ is requested to correspond to the stationary states of the system to be described, this if ensured through a variational optimization procedure.

2. Let the carrier density ρ be the real quantity defined above
$$\rho(\mathbf{x},t) = \Psi^\dagger(\mathbf{x},t)\Psi(\mathbf{x},t)$$
where $\rho(\mathbf{x},t)$, Ψ and $\Psi^\dagger$ are: unique-valued, continuous and twice-differentiable and obey the additional condition $\rho(\mathbf{x},t)|_{\text{space boundary}} = 0$.

3. Let the canonically conjugated variables be $(\mathbf{x},t)$ and $\Box\mathfrak{W} = iK\Box\ln\Psi = -iK\Box\ln\Psi^\dagger$, with $\Box = e^\mu\partial_\mu$ the space-time gradient operator.

4. The local energy description be ($\mathcal{E}_0$ is not a density)
$$K^2\frac{(\Box\Psi^\dagger)\cdot(\Box\Psi)}{\Psi^\dagger\Psi}c^2 = \mathcal{E}^2 - (Pc)^2 = (\mathcal{E}_0)^2 = \left(m_0c^2\right)^2$$
(in the case where an interaction is assumed to exist $(E-V)^2 - (Pc-eA)^2 = \left(m_0c^2\right)^2$) with the Euler–Lagrange (density of energy and constrain) function
$$J = K^2(\Box\Psi^\dagger)\cdot(\Box\Psi)c^2 - \left(m_0c^2\right)^2\Psi^\dagger\Psi$$
and perform the variational search for the extremum energy $\mathcal{E}$ (minimum of action for a stationary state system) $\delta J = 0$ to obtain from the standard variational approach the condition ($\kappa^2 = \hbar^2$)
$$K^2\left[\Psi^\dagger\left(\Box^2\Psi\right) + \left(\Box^2\Psi^\dagger\right)\Psi\right] = m_0c^2\Psi^\dagger\Psi$$
and then the equation for the auxiliary function Ψ (the Schrödinger-Klein-Gordon-like Equation (SKG), considering first V=A=0) is
$$\left[\hbar^2\left(\frac{\partial^2}{\partial t^2} - \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)c^2\right) - \left(m_0c^2\right)^2\right]\Psi = 0. \quad (11)$$

We must emphasize that in the relativistic (and in the non relativistic) case we obtain, through the Schrödinger optimization procedure, the Ψ (or $\Psi^\dagger$) **function which minimizes the action of the system**. A geometric factorization of the operator in the SKG equation transform it into a Dirac-like equation. In the next section we follow an alternative procedure which illustrates directly the meaning of the components of the Schrödinger Amplitude Function.

1.10. Linear form of the Amplitude Function Equation

We want to express the Schrödinger-Klein-Gordon equation (SKG, 11) in the linear form

$$\hat{H}_{linear}\psi = m_0 c^2 \psi. \tag{12}$$

Consider the simple case of the free carrier, that is, of the equation ($k^2 = (m_0 c/\hbar)^2$, $i^2 = -1$)

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi - \nabla^2\psi + k^2\psi = 0 \tag{13}$$

and write (following a procedure analog to that of Charlier, Bérard, Charlier and Fristot [9] to obtain a Schrödinger-like equation from the SKG)

$$\begin{aligned} \psi &= \sum_{a=1}^{m}\phi_a, \\ \frac{\partial\psi}{\partial x_\mu} &= \sum_{a=1}^{m} c_\mu^a \phi_a, \end{aligned} \tag{14}$$

$m = 8$ in order to have a faithful representation of both definitions and the coefficients are given, for example, by the matrix

$$c_\mu^a = ik\begin{pmatrix} +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 \\ +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 \\ +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 \end{pmatrix}$$

with these definitions we write the 4 equations

$$\frac{\partial}{\partial x_\mu}\sum_{a=1}^{m}\phi_a = -ik\sum_{a=1}^{m} c_\mu^a\phi_a$$

and use them in equation (11,13) to obtain:

$$\sum_\mu \frac{\partial}{\partial x_\mu}\sum_{a=1}^{8} c_\mu^a\phi_a = -ik\sum_{a=1}^{m}\phi_a$$

which is a linear form of the energy-momentum conservation equation The ϕ_a are not components of the wave function. Any structure of ψ will be given to the ϕ_a.(11).

If a representation of the same relation is given through the use of a set of real 8×8 or complex 4 × 4 matrices γ^μ, [$2\mathbb{C}(2)$] we obtain the Dirac equation. The Dirac equation is thus a faithful representation of the linearized form of the Schrödinger–Klein–Gordon equation.

Observe that we can now define a d'Alembertian operator

$$\Box = \gamma^\mu \partial\mu$$

which allows the generation of a Clifford algebra representation of the full geometry of space–time through the definition

$$\gamma(e^\mu) = \gamma^\mu$$

of the matrix representation of the geometric algebra of space–time. The γ^μ faithfully obey the same relations as the basis vectors e^μ. Representations of algebras are not unique, but they are related by similitude transformations.

1.11. First Order Equation as a Factorization-Projection

The deduced equations which are first order in the space–time derivatives can also be analyzed as a factorization and projection procedure. Consider again the equation for the optimized function to obtain the least action state of the system of independent equivalent carriers (here again $k^2 = (m_0 c/\hbar)^2$, $i^2 = -1$)

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi - \nabla^2\psi + k^2\psi = 0 \tag{15}$$

and propose the factorization of the operator in the Dirac sense ($\Box = \gamma^\mu\partial\mu$)

$$\begin{aligned} &(\Box + ik)(\Box - ik)\psi \\ &\qquad = (\gamma^\mu\partial\mu + ik)(\gamma^\mu\partial\mu - ik)\psi, \end{aligned} \tag{16}$$

defining the projected function

$$\Psi = (\gamma^\mu\partial\mu - ik)\psi \tag{17}$$

which obeys, by construction, the well known Dirac equation

$$(\gamma^\mu\partial\mu + ik)\Psi = 0 \tag{18}$$

showing that the, **optimized to obtain the least action, complete solution of the linear equation,** auxiliary function Ψ is a non–scalar function (which depends on the representation $\gamma(e_\mu) = \gamma^\mu$ of the geometry) containing information about the action and about the energy–momentum relationship through the $m_0 c$ and the space–time derivatives of the action.

This factorization is not unique in the case $\kappa = 0$, that is for a massless carrier field. This is further studied below. Here consider the factorization

$$(\partial^\mu\partial_\mu + m^2) = (D^\dagger + mi)(D - mi), \tag{19}$$

which requires that

$$-D^\dagger m + mD = 0 \text{ and } D^\dagger D = \partial^\mu\partial_\mu = \Box^2, \tag{20}$$

we can have then a set of choices, either:

1. any value of m and $D^\dagger = D$ (the standard Dirac operator $D_0 = \Box$);
2. or in the case where $m = 0$ the possibility $D^\dagger \neq D$ also become acceptable.

The basic requirement $D^\dagger D = DD^\dagger = \partial^\mu\partial_\mu$ limits the choices of D. Here they will be written in the

Lorentz invariant form $D_{(d,f)} = \Gamma^{\mu}_{(f)}\partial^{(d)}_{\mu}$, in order to show the relation to Dirac's original factorization in the simplest possible form. The $\Gamma^{\mu}_{(f)}$ are generalized (irreducible or reducible representation) Dirac γ^{μ} matrices. The limitation is so strong that the only possible choice is where the multi-vector $i\gamma^5$, which has the same action on all γ^{μ}, that is $i\gamma^5\gamma^{\mu} = -\gamma^{\mu}i\gamma^5$, is used (see Keller [3]).We define the differential

$$\partial^{(d)}_{\mu} = \Big\{ 1\cos(n+t^d_{\mu})\frac{\pi}{2} + i\gamma^5 \sin(n+t^d_{\mu})\frac{\pi}{2} \Big\}\partial_{\mu} \quad (21)$$

with n and t^d_{μ} integers, a choice which results in the simplest multi-vector. Here, to take the electron as a reference, we use $n = 1$.

Then, in a particular frame we have the 'diagonal' structure:

$$\partial^{(d)}_{\mu} = \begin{cases} \partial_{\mu} & \text{if } n+t^d_{\mu} \text{ are even,} \\ i\gamma^5\partial_{\mu} & \text{if } n+t^d_{\mu} \text{ are odd.} \end{cases} \quad (22)$$

The vectors, which are represented by the standard γ^{μ} matrices, correspond to an irreducible representation of $C_{1,3}$ and have been found to be useful for writing the wave equations of the fundamental family of leptons and quarks $(e^-_R, e^-_L, \nu_L, \{u_L, d_L;\ \text{color}\})$ of elementary particles. The electron requires a combination of two fields $e^- = (e^-_R, e^-_L)$ for the standard phenomenology of electroweak–color interactions. The case of the neutrino here presented is the simplest one of these structures.

1.12. Gauging of the Two Carriers System

We consider now the use of an auxiliary amplitude function Ψ approach for the study of the fundamental problem of the gauging of the two carriers system in order to represent their mutual interaction.

This is a very important analysis because it shows the role of the definition of a fundamental carrier as that physical object which will be observed if far from the rest of the objects. We transform the description of two carriers to that of two interacting carriers.

Consider a multi-carrier amplitude function $\Psi = \psi_1 \otimes \psi_2$ written as $\Psi = \Psi\left(\{x^{\mu}_c\};\ c = 1,2\right)$ indicating that a set of carriers can be described by an amplitude function depending in one set of coordinates x^{μ}_c for each carrier. For the physical description we use a set of operators like $p^c = i\hbar\partial^{(c)}_{\mu}$ and (external) energy contributions $V\left(\{x^{\mu}_c\}\right)$. The Γ^c_{μ} matrices of this representation are required to obey (upper index corresponds to carrier index, lower index to space–time coordinate components, Greek letters for space–time and Latin letters for space)

$$\begin{aligned} &\Gamma^c_{\mu}\Gamma^j_{\nu} + \Gamma^c_{\nu}\Gamma^j_{\mu} = 0, && c \neq j, \\ &\Gamma^c_{\mu}\Gamma^j_{\nu} + \Gamma^c_{\nu}\Gamma^j_{\mu} = 2\eta_{\mu\nu}, && c = j, \\ &\Gamma^c_{\mu}\cdot\Gamma^j_{\nu} = \delta^{cj}\eta_{\mu\nu}, && \end{aligned} \quad (23)$$

and the (per carrier) bi-vectors, etc., can be defined

$$\Gamma^1_{i0} = \Gamma^1_i\Gamma^1_0,\ \Gamma^2_{i0} = \Gamma^2_i\Gamma^2_0, \quad \text{etc.}. \quad (24)$$

These objects commute

$$\Gamma^1_{i0}\Gamma^2_{j0} = \Gamma^2_{j0}\Gamma^1_{i0}. \quad (25)$$

We define the system's pseudo-scalar as the product of the $4+4 = 8$ vectors associated with the two carriers. We also define $\Gamma^i_5 = \Gamma^i_0\Gamma^i_1\Gamma^i_2\Gamma^i_3$ for each carrier and the global

$$\Gamma_5 = \Gamma^1_5\Gamma^2_5. \quad (26)$$

In this presentation the allowed set of algebraic solutions Ψ for the system are constructed from pairs of double projections, one double projector for each carrier, and have a matrix structure which can be represented by an equivalent column matrix, as the outer product of the representative column matrices for each carrier. For each carrier the column matrix has four entries as a result of the use for each carrier of the double projection (mass projector uses the time-like vector γ_0 spin plane$\Leftrightarrow \gamma_{12}$) $\frac{1}{2}(1\pm\gamma_0)\,\frac{1}{2}(1\pm i\gamma_{12})$

$$\begin{pmatrix}1\\0\\0\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \quad \begin{pmatrix}0\\1\\0\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \quad \text{etc..}$$

The first column describes carrier 1 and the second carrier 2, in that case for the two carriers system with zero total spin

$$\psi\Gamma^1_5\Gamma^1_{30} = -\psi\Gamma^2_5\Gamma^2_{30}$$

and then we can, for the two particle auxiliary function ψ define

$$\psi = \psi\Gamma^1_5\Gamma^1_{30}\Gamma^2_5\Gamma^2_{30}$$

a procedure equivalent to the use of the joint double projector

$$\psi = \psi\frac{1}{2}\left(1 + \Gamma^1_5\Gamma^1_{30}\Gamma^2_5\Gamma^2_{30}\right).$$

The extension allows the definition of a set of double projectors from $S_{\uparrow\uparrow}$

$$S_{\uparrow\uparrow} = \frac{1}{2}\left(1 - \Gamma^1_5\Gamma^1_{30}\Gamma^2_5\Gamma^2_{30}\right), \qquad S^2_{\uparrow\uparrow} = S_{\uparrow\uparrow},$$

and its factorization

$$S_g = S_{\uparrow\uparrow}\,\Gamma^1_5\Gamma^1_{30} = S_{\uparrow\uparrow}\,\Gamma^2_5\Gamma^2_{30} = \frac{1}{2}\left(\Gamma^1_5\Gamma^1_{30} + \Gamma^2_5\Gamma^2_{30}\right),$$

such that

$$S_g^2 = -S_{\uparrow\uparrow},$$

which allows us to obtain, by projection with $\frac{1}{2}\left(1 \pm i\Gamma_{12}^1\right)\frac{1}{2}\left(1 \pm i\Gamma_{12}^2\right)$ from a general solution, the set of all four possible total **spin** choices: $+1, -1, 0, 0$. We use the zero total spin case only.

Defining the space–time gradient operators

$$\Box^{(1)} \equiv \Gamma_\mu^1 \frac{\partial}{\partial x_\mu^1}, \qquad \Box^{(2)} \equiv \Gamma_\mu^2 \frac{\partial}{\partial x_\mu^2},$$

rewriting the free carrier equation

$$i\hbar\Box^{(c)}\psi = m_c\psi \quad \text{as} \quad \frac{i\hbar\Box^{(c)}}{m_c}\psi = \psi,$$

and using the properties

$$\psi\Gamma_0^c = \psi \quad \text{and} \quad \psi i\Gamma_{12}^c = \pm\psi,$$

we can formally sum for the two 'free', non-interacting carriers, to obtain

$$\hbar\left(\frac{\Box^{(1)}}{m_1} + \frac{\Box^{(2)}}{m_2}\right)\psi(x)\left(\Gamma_{012}^1 - \Gamma_{012}^2\right) = 2\psi(x) \quad (27)$$

where we have used the joint coordinates

$$x = x^1 + x^2 = \Gamma_\mu^1 (x^1)^\mu + \Gamma_\mu^2 (x^2)^\mu.$$

The (projected by spin) amplitude function ψ corresponding to the description of two **independent** carriers can, by definition, be factorized

$$\psi = \psi^1\psi^2,$$

to obtain from (27), using the projector $S_{\uparrow\downarrow}$ to ensure the correct spin for each carrier

$$\hbar\left(\frac{\Box^{(1)}}{m_1} + \frac{\Box^{(2)}}{m_2}\right)\psi^1\psi^2 S_{\uparrow\downarrow}\left(\Gamma_5^1\Gamma_3^1 - \Gamma_5^2\Gamma_3^2\right) = 2\psi^1\psi^2 S_{\uparrow\downarrow}. \quad (28)$$

Following our procedure of redistributing the energy in a gauge-free form we transform (27) by a phase factor rotation. This requires us to understand the gauging properties of the equation by studying its behavior under Lorentz transformations. Below $R^c(x,t) = \exp(\theta^{\mu\nu}(x,t)\Gamma_{\mu\nu}^c/2)$ is a generator of a Lorentz transformation, in multivector form.

For the transformation of the basic equations above into the gauged equations, where the action is shared through an interaction, consider a pair of Lorentz transformations R^1 and R^2 acting as

$$R^c\Gamma_\mu^c\widetilde{R^c} = \Gamma_\mu'^c$$

and their action on the amplitude function

$$\psi(x) \longmapsto \psi'(x) \equiv R^1R^2\psi\left(\widetilde{R^2}\widetilde{R^1}xR^1R^2\right),$$

which is induced by the application of R^1R^2 to (27) or (28). For the analysis of the two carriers system consider the particular case

$$R^1\Gamma_{12}^1\widetilde{R^1} = \Gamma_{12}^1, \quad R^2\Gamma_{12}^2\widetilde{R^2} = \Gamma_{12}^2$$

where by definition of space–time volume

$$R^1\Gamma_5\widetilde{R^1} = R^2\Gamma_5\widetilde{R^2} = \Gamma_5,$$

such that R^1 and R^2 are Lorentz boosts in the γ_3 direction, with opposite signs and rotations in the γ_{12} plane, also with opposite signs. With this choice $x \to x' = \widetilde{R^2}\widetilde{R^1}xR^1R^2$ corresponds to induced separation of the carriers in the z direction with a synchronized rotation of the spin plane (perpendicular to z). This can be compensated if the new amplitude function ψ' obeys a gauged equation

$$\hbar\left(\frac{\Box^{(1)}}{m_1} + \frac{q_1}{m_1}A^{(1)} + \frac{\Box^{(2)}}{m_2} + \frac{q_2}{m_2}A^{(2)}\right) \times \psi'(x')\left(\Gamma_{012}^1 - \Gamma_{012}^2\right) = 2\psi'(x'). \quad (29)$$

For an observer the new situation is that of ψ' describing two interacting carriers in relative motion, with their spins having a relative additional rotation. The carriers have acquired kinetic energy and generated a reciprocal interaction proportional to q_c. If the total energy is required to be constant for all values of the joint position x, then the vector interaction fields $A^{(c)}$ obey the Maxwell equations. (See [3] where spin and electromagnetic interactions are analyzed). The final step is to define the interaction of the second carrier onto the first as a multivector $A^{(1)}$ and, conversely, $A^{(2)}$ from the definition of interaction fields, and to substitute these terms in (29) to obtain a pair-wise interaction term.

1.12..1 Electroweak Interaction of Two Particles

For the case of the chiral massless field of the neutrino the procedure follows as above changing $\Gamma_0 \to H = i\Gamma_5$, $m_c = 0$ and $\Box^{(c)} \to D^{(c)}$ in the previous section. Then, for the study of the weak interaction follow:
define the space–time gradient operators

$$D^{(1)} = \Box_W^{(1)} \equiv \Gamma_0^1\frac{\partial}{\partial x_0^1} - H\nabla^{(1)},$$

$$D^{(2)} = \Box_W^{(2)} \equiv \Gamma_0^2\frac{\partial}{\partial x_0^2} - H\nabla^{(2)}$$

where

$$\nabla^{(c)} = \Gamma_i^c\frac{\partial}{\partial x_i^c}, \; i = 1,2,3$$

and $\nabla^{(c)}$ is a dual vector. Rewrite the free carrier equation for the chiral massless case

$$i\hbar\Box_W^{(c)}\psi = 0$$

and define the chirality and spin projection properties of the wave function ψ

$$\psi i\Gamma_5^c = \psi \quad \text{and} \quad \psi i\Gamma_{12}^c = \pm\psi,$$

then formally sum for the two 'free', non-interacting carriers, to obtain

$$\hbar\left(D^{(1)} + D^{(2)}\right)\psi(x)\left(\Gamma_{03}^1 - \Gamma_{03}^2\right) = 0, \qquad (30)$$

notice that the post factor here is a bi-vector, and that we have used the joint coordinates

$$x = x^1 + x^2 = \Gamma_\mu^1 (x^1)^\mu + \Gamma_\mu^2 (x^2)^\mu.$$

Again the (projected by spin and chirality) amplitude function ψ corresponding to the description of two **independent** carriers can, by definition, be factorized

$$\psi = \psi^1\psi^2$$

to obtain from (30), using the projector $S_{\uparrow\downarrow}$ to ensure the correct spin for each carrier, the amplitude function ψ^c

$$\hbar\left(D^{(1)} + D^{(2)}\right)\psi^1\psi^2 S_{\uparrow\downarrow} \times \left(\Gamma_0^1\Gamma_3^1 - \Gamma_0^2\Gamma_3^2\right) = 0, \quad (31)$$

now following our procedure of redistributing the energy in a gauge-free form transform (30) by a phase factor rotation, using a pair of Lorentz transformations R^1 and R^2 acting as

$$R^c\Gamma_\mu^c\widetilde{R^c} = \Gamma_\mu'^c, \quad \psi(x) \longmapsto \psi'(x) \equiv R^1R^2\psi\left(\widetilde{R^2}\widetilde{R^1}xR^1R^2\right),$$

induced by the application of R^1R^2 to (30). Consider the particular case

$$R^1\Gamma_{12}^1\widetilde{R^1} = \Gamma_{12}^1, \quad R^2\Gamma_{12}^2\widetilde{R^2} = \Gamma_{12}^2,$$

and by definition of space–time volume

$$R^1\Gamma_5\widetilde{R^1} = R^2\Gamma_5\widetilde{R^2} = \Gamma_5,$$

such that R^1 and R^2 are Lorentz boosts in the γ_3 direction, with opposite signs and rotations in the γ_{12} plane, also with opposite signs. With this choice $x \to x' = \widetilde{R^2}\widetilde{R^1}xR^1R^2$ corresponds to an induced separation of the carriers in the z direction with a synchronized rotation of the spin plane (perpendicular to z). This can be compensated if the new amplitude function ψ' obeys a gauged equation

$$\hbar\left(D^{(1)} + q_1^W B^{(1)} + D^{(2)} + q_2^W B^{(2)}\right)\psi'(x') \times \left(\Gamma_{03}^1 - \Gamma_{03}^2\right) = 0. \quad (32)$$

For an observer the new situation is that of ψ' describing two interacting carriers in relative motion, with their spins having a relative additional rotation. The carriers have acquired kinetic energy and generated a reciprocal interaction proportional to q_c^W. If the total energy is required to be constant for all values of the joint position x, then the dual-vector interaction $B^{(c)}$ should, in principle, obey the (dual of) Maxwell equations. Here the ψ^c should be chiral and both of the same chirality.

But the massless fields represent a carrier traveling at the speed of light, then only for zero separation of the interacting carriers the (retarded) $B^{(c)}$ fields would be different from zero, then the weak interaction is geometrically constrained to be local.

The chiral field $\psi^{(1)}$ can correspond to a neutrino and $\psi^{(2)}$ to the left handed projection of an electron field, projection which obeys $\psi^{(2)}i\Gamma^2 = \psi^{(2)}$. For the electron we write $\psi^{(e)} = \psi_L^{(e)} + \psi_R^{(e)}$ with, here, $\psi^{(2)} = \psi_L^{(e)}$.

2. Theoretical Description of the Neutrino and of the Electron in START

First it is convenient to summarize the resulting model for the electron when, in the description of a field, the basic properties of mass, spin, and charge are included. The action distribution is given a set of geometrical symmetries by requiring that the field corresponds to the most elementary field in START.

The rest energy of the field, resulting from an integration where $\int_V \rho dx = 1$, is m_0c^2, and obeys a wave equation in the START geometry, the simplest representation corresponds to spin $\hbar/2$, its coupling to receive action from another field is given by e, the ratio e/m_0 corresponding to the rate of change of energy with action per unit energy of the original field. Correspondingly, when action is given to other fields the strength of this action is also proportional to e. When work can be done on or by the field, by absorbing or by creating a quanta of the interaction field (redistributing the action), the emitted or absorbed energy per elementary action corresponds to a change of spin equal to $\hbar$, or to a corresponding change in angular momentum of orbital origin. We have to make a clear distinction between **actual** energies given to the field and **relative** energies which are described by a gauge field, even if in both cases the description of energy demands the definition of a frame of reference with respect to which one carrier acquires more energy at the expense of the energy of the environment.

The elementary fields described by the model are required to be created or annihilated by at least pairs of fields with mutually canceling symmetry properties. Then a collection of fields, where no other fields are present which can cancel the symmetry properties of this collection, will correspond to stable matter. The distributions will exist in space but can only be created and annihilated in units of action, that is that the change in spin has to be a multiple of $\hbar/2$ and the change in energy with respect to a reference observer must correspond to $h\nu$. When fields at rest in a frame of reference are created (annihilated) the energy of the distribution is $h\nu = m_0 c^2$. These conditions originate the notion of carrier within the theory, a concept that will be even closer to the classical limit if the distribution domain is small compared to the distances involved in the global experimental observation. We can change the form of the action distribution in a continuous form, but we can not change the existence of the distribution except in a quantized form. At the same time it is now clear that at the level of the elementary fields (first) quantization corresponds to a type of description considering processes where the properties of the distributions can be changed, and second quantization to processes where distributions are created or annihilated.

2.1. The Local Structure of the Action Density

The basic postulate for the theory of matter and interactions fields here presented is the existence of a distribution of action, with certain geometrical characteristics, in a region of space–time.

For **massless** fields as in the case of the neutrino a local system of vectors, at each point, defines the geometric characteristics: a vector a in direction e_4 for the intensity of action; a vector in direction e_0 for the (unavoidable) time direction. The rate of change of a with respect to time being the density of energy of the massless field $\varepsilon = \rho(X)h\nu$. The remaining three directions are internally defined in this case because all massless fields are chiral, including the interaction fields. One of the space-like directions corresponds to the direction of propagation of the massless field, with momentum $p = \rho(X)h/\lambda$ parallel to velocity. The remaining two space-like directions define the plane of the spin, perpendicular by definition to the propagation direction. This is the local structure of the action density corresponding to massless matter and interaction fields, this structure is by no means trivial and is reflected in the gauging properties of the description of the field. We should stress here that every point of the distribution is given the same set of local properties. The gauge freedom can only change the relative values of these sets of vectors at different space–time points with the condition that the local structure has to be respected. Chirality and the relation $\varepsilon'/p' = c$ are basic features of the description of the massless fields, the primed energy momentum components related to the unprimmed ones by a Lorentz transformation.

For **massive** fields the local structure is similar except that the set of three space-like vectors, even if their directions are related to momentum and spin, offers now the additional gauge freedom of the direction of the spin plane being orientable, independently of the momentum direction. In fact, for the electron there is at least one local frame of reference where $p = 0$ and, according to our principles above, $\partial a/\partial t = m_0 c^2$ also $\left|(dx^4)^2\right| = (dx^0)^2$. For the massive fields, the electron being our example here, we then have that for a general observer there is a current $\mathbf{j}$ which is related to the local frame where the carrier field is at rest (frame vectors $\left\{\gamma_u^{(0)}\right\}$) through a Lorentz transformation

$$\mathbf{j} = j^\mu \gamma_\mu = \rho v^\mu \gamma_\mu = R\rho\gamma_0^{(0)} R^{-1}, \qquad (1)$$

showing that to each point of the distribution corresponds a current $\mathbf{j}$, this being one of the basic properties of the matter field. The direction of the current and the direction of the momentum are two different quantities, the current being related to the transformation of the local frame of reference and the momentum to the rate of change in space of the action distribution. In a similar form the spin $\mathbf{s}$ is related to a spin plane in the local frame of reference $\gamma_{12}^{(0)}$ by the same Lorentz transformation

$$\mathbf{s} = \rho s^{\mu\nu}\gamma_{\mu\nu} = R\rho\gamma_{12}^{(0)} R^{-1}. \qquad (2)$$

Idealized currents can be considered for computational purposes. The best known example being perhaps that of a plane wave, even if in practice no actual currents of matter or radiation can be approximated by such a current, except for a very small region of space, and, moreover, only in the case where the actual current corresponds to a (steady) current of matter or radiation, consisting of a large number of matter or radiation units.

Because our description of matter and its interactions has as starting point a definition which corresponds to a definite picture of nature, there is a temptation to interpret several consequences of the theory as physical descriptions of nature, even if in many cases they are only one form of description among many, or if they correspond to an approximate description useful for calculations but not to a comprehensive description of the different phenomena. In the other extreme we can analyze the different features of the theory in relation to well established mathematical models. Our theory reproduces the mathematical structure of density functional theory and the mathematical structure of quantum mechanics, then the calculational procedures can be carried on, having either of both presentations as a guide, given that the mathematical structure allows it. A basic difference is that we are using a continuous model for matter: a distribution of action where every point is endowed of a geometrical feature.

3. Densities and Currents of the Electron

We have obtained the (electromagnetic field) gauged equation for the action amplitude auxiliary wave function ψ of an electron:

$$\gamma^{\eta}\left(\frac{\hbar}{i}\frac{\partial}{\partial x^{\eta}}+\frac{e}{c}A_{\eta}\right)\psi=-mc\psi \qquad (1)$$

in terms of the $A = \gamma^{\eta}A_{\eta}$ the electromagnetic potential vector. For the neutrino $m = 0$.

In complex geometry the *complex conjugate* $\overline{\mathcal{M}}$ of a multi-vector $\mathcal{M}$ is the complex conjugate of each coefficient of each multi-vector basis in the sum representing $\mathcal{M}$. That is given ($i < j < k < m$)

$$\mathcal{M}=IM+\gamma_{i}M^{i}+\gamma_{ij}M^{ij} + \gamma_{ijk}M^{ijk}+\gamma_{ijk\mathbf{m}}M^{ijkm}+..., \qquad (2)$$

$$\overline{\mathcal{M}}=I\overline{M}+\gamma_{i}\overline{M}^{i}+\gamma_{ij}\overline{M}^{ij} + \gamma_{ijk}\overline{M}^{ijk}+\gamma_{ijk\mathbf{m}}\overline{M}^{ijkm}+... \qquad (3)$$

where $\overline{M}^{i_1i_2\ldots i_n}$ denotes the complex conjugate of $M^{i_1i_2\ldots i_n}$. Similarly define the *complex reverse* $\mathcal{M}^{+}$ of a complex multi-vector $\mathcal{M}$ as the complex conjugate of $\mathcal{M}$ and the reverse of the order of all products of vectors generating the basis multi-vectors, for (2) above

$$\mathcal{M}^{+}=I\overline{M}+\gamma_{i}\overline{M}^{i}+\gamma_{ji}\overline{M}^{ij} + \gamma_{kji}\overline{M}^{ijk}+\gamma_{mkji}\overline{M}^{ijkm}+... \qquad (4)$$

If $\mathcal{M}^{+}=\mathcal{M}$ then $\mathcal{M}$ is Hermitian, and if $\mathcal{M}^{+}=\overline{\mathcal{M}}$ then $\mathcal{M}$ is unitary. The *complex scalar product* defined above becomes

$$\langle\mathcal{M},\mathcal{B}\rangle=\left(\mathcal{M}^{+}\mathcal{B}\right)_{0} \qquad (5)$$

again $(\mathcal{M}^{+}\mathcal{B})_0$ designates the (0-vector or) scalar part of $\mathcal{M}^{+}\mathcal{B}$. See [12].

We use now the Fock and Ivanenko equation for curved space introducing the Fock-Ivanenko $\mathbf{\Omega}(a)$ bi-vector fields coefficients. Locally the Dirac equation for curved space becomes the gauged with non-commuting fields

$$\gamma^{\eta}\left(\frac{\partial}{\partial x^{\eta}}-\mathbf{\Omega}_{\eta}+\frac{ie}{\hbar c}A_{\eta}\right)\psi=-\frac{imc}{\hbar}\psi. \qquad (6)$$

Rewriting the covariant derivative $D = \gamma^{\mu}\Box_{\mu}$ of a multi-vector $\mathbf{M}$ in the form

$$\left(\frac{\partial}{\partial x^{\eta}}-\mathbf{\Omega}_{\eta}\right)[\mathbf{M}]=\Box_{\eta}[\mathbf{M}]-[\mathbf{M}]\,\mathbf{\Omega}_{\eta}. \qquad (7)$$

We write (6) as

$$\gamma^{\eta}\Box_{\eta}\psi-\gamma^{\eta}\psi\mathcal{G}_{\eta}=-\frac{imc}{\hbar}\psi \qquad (8)$$

where the gauging fields (other terms could be added as discussed above, not included here to avoid the discussion of their chiral nature and their breaking of the chiral symmetry)

$$\mathcal{G}_{\eta}=\mathbf{\Omega}_{\eta}-\frac{ie}{\hbar c}A_{\eta}I. \qquad (9)$$

We now follow the usual steps to obtain the several currents. From the complex reverse of (6), one gets an equation for ψ^{+}

$$\left(\Box_{\eta}\psi^{+}\right)\gamma^{\eta}-\mathcal{G}_{\eta}^{+}\psi^{+}\gamma^{\eta}=\frac{imc}{\hbar}\psi^{+}. \qquad (10)$$

The set of Ω_{η} are real bi-vectors (that is with real coefficients) and i appears in the vector gauge field A

$$\mathcal{G}_{\eta}^{+}=-\mathcal{G}_{\eta} \qquad (11)$$

and

$$\left(\Box_{\eta}\psi^{+}\right)\gamma^{\eta}+\mathcal{G}_{\eta}^{+}\psi^{+}\gamma^{\eta}=\frac{imc}{\hbar}\psi^{+}. \qquad (12)$$

To obtain the vector current we multiply (12) from the right by ψ and multiply (8) from the left by ψ^{+} and sum the resulting equations to obtain the η component of the current equations in curved space–time

$$\left(\Box_{\eta}\psi^{+}\right)\gamma^{\eta}+\psi^{+}\gamma^{\eta}\Box_{\eta}\psi + \mathcal{G}_{\eta}\left(\psi^{+}\gamma^{\eta}\psi\right)-\left(\psi^{+}\gamma^{\eta}\psi\right)\mathcal{G}_{\eta}=0, \qquad (13)$$

as the scalar part of (13). As the scalar part is symmetric in the product of any two Clifford numbers the $()_0$ projection (scalar part) of the two last terms will cancel out to obtain

$$\left(\left(\Box_{\eta}\psi^{+}\right)\gamma^{\eta}\psi+\psi^{+}\gamma^{\eta}\Box_{\eta}\psi\right)_{0}=0. \qquad (14)$$

For the index free multi-vector ψ the covariant derivative $\psi_{;\eta}$ is equal to $\Box_{\eta}\psi$. Furthermore by construction $\gamma^{\alpha}_{;\eta}=0$. Then (14) becomes

$$\left(\psi^{+}_{;\eta}\gamma^{\eta}\psi+\psi^{+}\gamma^{\eta}_{;\eta}\psi+\psi^{+}\gamma^{\eta}\psi_{;\eta}\right)_{0}=0,$$

or

$$\left(\left(\psi^{+}\gamma^{\eta}\psi\right)_{0}\right)_{;\eta}=0. \qquad (15)$$

This conservation law allows the definition of a current $\mathbf{J} = j^{\mu}\gamma_{\mu}$, where

$$J^{\eta}=e\left(\psi^{+}\gamma^{\eta}\psi\right)_{0}, \qquad (16)$$

can be interpreted as a conserved current satisfying

$$J^{\eta}_{;\eta}=0. \qquad (17)$$

A multi-vector interpretation of (16) is obtained from the use of the fact that one may exchange the order of the product of two Clifford numbers before projecting out the scalar component without changing the result. Then

$$\left(\psi^{+}\gamma^{\eta}\psi\right)_{0}=\left(\gamma^{\eta}\psi^{+}\psi\right)_{0}=\left\langle\gamma^{\eta},\psi^{+}\psi\right\rangle,$$

allowing the definition of the current components

$$J^\eta = e\left\langle \gamma^\eta, \psi^+\psi \right\rangle, \tag{18}$$

considering $\langle \gamma^\eta, \gamma_\alpha \rangle = \delta^\eta_\alpha$, as the vector part of the multi-vector product $\psi\psi^+$. That is

$$\psi\psi^+ = sI + \frac{J}{e} + \mathbf{M} + K + p\gamma_5 \tag{19}$$

with $\mathbf{M}$ an angular momentum field and K an axial current.

The Gordon decomposition is an analysis of the bi-vector components of $\psi\psi^+$ (Gordon 1928).

Following the Gordon analysis the current

$$j^\eta = \psi^+\gamma^\eta\psi \tag{20}$$

is reinterpreted in terms of the gauged fields

$$\psi = \frac{i\hbar}{mc}\left(\gamma^\nu \Box_\nu \psi - \gamma^\nu \psi \mathcal{G}_\nu\right)$$

or considering again covariant derivatives

$$\psi = \frac{i\hbar}{c}\left(\gamma^\nu \psi_{;\nu} - \gamma^\nu \psi \mathcal{G}_\nu\right). \tag{21}$$

From the complex reverse of this last equation we obtain for ψ^+

$$\psi^+ = \frac{i\hbar}{mc}\left(\psi^+_{;\nu}\gamma^\nu + \mathcal{G}_\nu \psi^+ \gamma^\nu\right) \tag{22}$$

to obtain

$$\psi^+\gamma^\eta\psi = \frac{i\hbar}{2mc}\left[\psi^+\gamma^\eta\gamma^\nu\psi_{;\nu} - \psi^+\gamma^\eta\gamma^\nu\psi\mathcal{G}_\nu - \psi^+_{;\nu}\gamma^\nu\gamma^\eta\psi - \mathcal{G}_\nu\psi^+\gamma^\nu\gamma^\eta\psi\right].$$

The main point of the analysis arises from the basic geometrical definition $\gamma^\eta\gamma^\nu = \gamma^{\eta\nu} + g^{\eta\nu}I$, to rewrite (20) in the form

$$\begin{aligned}\psi^+\gamma^\eta\psi = \frac{i\hbar}{2mc}\Big[&\left(\psi^+\gamma^{\eta\nu}\psi_{;\nu} + \psi^+_{;\nu}\gamma^{\eta\nu}\psi\right) \\ &+ g^{\eta\nu}\left(\psi^+\psi_{;\nu} - \psi^+_{;\nu}\psi\right) \\ &- g^{\eta\nu}\left(\psi^+\psi\mathcal{G}_\nu + \mathcal{G}_\nu\psi^+\psi\right) \\ &- \left(\psi^+\gamma^{\eta\nu}\psi\mathcal{G}_\nu - \mathcal{G}_\nu\psi^+\gamma^{\eta\nu}\psi\right)\Big].\end{aligned} \tag{23}$$

Since $\gamma^{\eta\nu}_{;\nu} = 0$, the first pair of terms on the right-hand side of (23) may be combined into a single terms $(\psi^+\gamma^{\eta\nu}\psi)_{;\nu}$. In addition, if we project out the scalar component of both sides of (23), the last pair of terms will cancel out. We then have

$$\begin{aligned}J^\eta = e\left(\psi^+\gamma^\eta\psi\right)_0 = &\frac{ie\hbar}{2mc}\left(\left(\psi^+\gamma^{\eta\nu}\psi\right)_{;\nu}\right)_0 \\ &+ \frac{ie\hbar}{2mc}g^{\eta\nu}\Big[\left(\psi^+\psi_{;\nu} - \psi^+_{;\nu}\psi\right)_0 \\ &- \left(\psi^+\psi\mathcal{G}_\nu + \mathcal{G}_\nu\psi^+\psi\right)_0\Big].\end{aligned} \tag{24}$$

Following the Gordon interpretation the first term on the right hand side of (24) can be interpreted as the *proper current*:

$$\begin{aligned}J^\eta_{\mathrm{INT}} &= \frac{ie\hbar}{2mc}((\psi^+\gamma^{\eta\nu}\psi)_{;\nu})_0 \\ &= \frac{ie\hbar}{2mc}\left(\left(\gamma^{\eta\nu}\psi\psi^+\right)_{;\nu}\right)_0 = \mathbf{M}^{\nu\eta}_{;\nu}.\end{aligned} \tag{25}$$

It is fundamental that J^η_{INT} is covariant and satisfies a continuity equation.

Above the (antisymmetric) bi-vector $\mathbf{M}$

$$\mathbf{M} = M^{\nu\eta}\gamma_{\nu\eta} = \frac{ie\hbar}{2mc}\left(\gamma^{\nu\eta}\psi\psi^+\right)_0\gamma_{\nu\eta} \tag{26}$$

is defined. It obeys

$$J^\eta_{\mathrm{INT};\eta} = M^{\nu\eta}_{;\nu;\eta} = 0. \tag{27}$$

Gordon also defined the second term on the right hand side of (24) as a *convection current* J^η_{CONV}:

$$\begin{aligned}J^\eta_{\mathrm{CONV}} = \frac{ie\hbar}{2mc}g^{\eta\nu}\Big[&\left(\psi^+\psi_{;\nu} - \psi^+_{;\nu}\psi\right)_0 \\ &-2\left(\mathcal{G}_\nu\psi^+\psi\right)_0\Big]\end{aligned} \tag{28}$$

which separately satisfies a continuity equation given that

$$J^\eta_{;\eta} = J^\eta_{\mathrm{INT};\eta} = 0. \tag{29}$$

The proper current J^η_{INT} is an internal current of the action distribution allowing the $M^{\nu\eta}$ to be interpreted as the electric and magnetic dipole moments of it. A reference system can be defined where the principal components are

$$P_z = -M^{30} = \frac{ie\hbar}{2mc}\left(\gamma^{30}\psi\psi^+\right)_0 \tag{30}$$

and

$$M_z = -M^{12} = \frac{ie\hbar}{2mc}(\gamma^{12}\psi\psi^+)_0 = \frac{e}{mc}s_z \tag{31}$$

with s_z the z-th component of the spin of the distribution.

The full set of currents is

$$\begin{aligned}e\psi\psi^+ = IA + J^\mu\gamma_\mu - \frac{imc}{\hbar}M^{\mu\nu}\gamma_{\mu\nu} \\ + i\gamma_5 K^\mu\gamma_\mu + p\gamma_5\end{aligned} \tag{32}$$

where the (imaginary) axial current is mentioned above. The auxiliary amplitude wave function ψ is not an eigenfunction of $i\gamma_5$ then the last two terms do not correspond to gauge-free conserved currents, only the left handed part or the right handed part separately.

We have mentioned above the general gauge transformation for ψ. We have a non-abelian gauge group (this is more clearly seen with the analysis of Snygg 1997, chapter 8 [12]). If all physically measurable information stored in the wave function ψ is also stored in the product $\psi\psi^+$, then the wave function ψ must be considered equivalent to the wave function $\psi\mathcal{S}$ where $\mathcal{S}$ is any differentiable unitary Clifford number containing the information compatible with our Principle of Choice of Acceptable Descriptions: The composite physical bodies can be described in any form which corresponds to the same total density of action.

Then substituting

$$\psi' = \psi\mathcal{S} \tag{33}$$

where ψ is a solution of (8). The ψ' are solutions of

$$\gamma^\eta \Box_\eta \psi' - \gamma^\eta \psi' \mathcal{G}'_\eta = -\frac{imc}{\hbar}\psi' \tag{34}$$

where, by a gauge transformation

$$\mathcal{G}'_\eta = \mathcal{S}^+ \mathcal{G}_\eta \mathcal{S} - (\Box_\eta \mathcal{S}^+)\mathcal{S} \tag{35}$$

then, as $\Box_\nu I = 0$

$$\left(\Box_\nu \mathcal{S}^+\right)\mathcal{S} = \Box_\nu\left(\mathcal{S}^+\mathcal{S}\right) - \mathcal{S}^+\Box_\nu\mathcal{S} = -\mathcal{S}\left(\Box_\nu\mathcal{S}\right) \tag{36}$$

and

$$\Box_\nu \mathcal{G}'_\eta - \mathcal{G}'_\eta \mathcal{G}'_\nu = \mathcal{S}^+\left(\Box_n \mathcal{G}_\eta - \mathcal{G}_\eta \mathcal{G}_\nu\right)\mathcal{S} + \left(\Box_\nu \mathcal{S}^+\right)\mathcal{G}_\eta\mathcal{S} + \left(\Box_\eta \mathcal{S}^+\right)\mathcal{G}_\nu\mathcal{S} - \left(\Box_\nu\Box_\eta\mathcal{S}^+\right)\mathcal{S} \tag{37}$$

from this relation it follows that

$$\Box_\nu \mathcal{G}'_\eta - \Box_\nu \mathcal{G}'_\nu + \mathcal{G}'_\nu \mathcal{G}'_\eta - \mathcal{G}'_\eta \mathcal{G}'_\nu = \mathcal{S}^+\left(\Box_\nu \mathcal{G}_\eta - \Box_\eta \mathcal{G}_\nu + \mathcal{G}_\nu\mathcal{G}_\eta - \mathcal{G}_\eta\mathcal{G}_\nu\right)\mathcal{S} - \left(\left(\Box_\nu\Box_\eta - \Box_\eta\Box_\nu\right)\mathcal{S}^+\right)\mathcal{S}$$

and from the definition of the Ricci curvature

$$\Box_\nu \mathcal{G}'_\eta - \Box_\eta \mathcal{G}'_\nu + \mathcal{G}'_\nu \mathcal{G}'_\eta - \mathcal{G}'_\eta \mathcal{G}'_\nu = \mathcal{S}^+\bigg(\Box_\nu \mathcal{G}_\eta - \Box_\eta \mathcal{G}_\nu + \mathcal{G}_\nu\mathcal{G}_\eta - \mathcal{G}_\eta\mathcal{G}_\nu - \frac{1}{2}\mathcal{R}_{\nu\eta}\bigg)\mathcal{S}. \tag{38}$$

In particular if $\mathcal{G}_\eta = \mathbf{\Omega}_\eta - (ie/\hbar c)\,A_\eta I$, then

$$\Box_\nu \mathcal{G}_\eta - \Box_\eta \mathcal{G}_\nu + \mathcal{G}_\nu\mathcal{G}_\eta - \mathcal{G}_\eta\mathcal{G}_\nu - \frac{1}{2}\mathcal{R}_{\nu\eta} = -\frac{ie}{\hbar c}F_{\nu\eta}I. \tag{39}$$

Since $\mathcal{S}$ commutes with I, it then follows that the left hand side of (38) is also equal to $-\,(ie/\hbar c)\,F_{\nu\eta}I$. But there are solutions of (39) more general than $\Omega_\eta - (ie/\hbar c)\,A_\eta I$, in particular the ones we have used to study the electroweak interaction, although as we mentioned above they break the chiral symmetry.

The curvature equation has been extended to include the geometric field intensities

$$\gamma^\eta\left(\Box_\nu \mathcal{G}_\eta - \Box_\eta \mathcal{G}_\nu + \mathcal{G}_\nu\mathcal{G}_\eta - \mathcal{G}_\eta\mathcal{G}_\nu\right) = \left(-\frac{1}{2}R_{\nu\eta} - \frac{ie}{\hbar c}F_{\nu\eta}\right)\gamma^\eta,$$

as already noted by Fock 1929.

This set of currents describe the symmetry constrains defining the electron as a stable carrier.

4. The Electroweak Interaction of the Neutrino–Electron Field

In order to understand the interactions of a field we have to consider first that the name ‘interactive’ itself expresses a relative property, the relation existing between two carrier fields which should have some gauge freedom of description allowing them to be considered together, as expressed in one of the sections above. We saw the case of the electromagnetic interaction. In the description of the gauging of the electron field we already mentioned the possibility of considering bi-vector valued phases or pseudo-scalar valued phases. The bi-vector-valued phases were found long time ago, by Fock and Ivanenko, to correspond to gravitational interactions. Here we shall describe the electroweak interaction as an extension of the electromagnetic case. We shall show that it corresponds to a pseudo-scalar valued phase. For this purpose we first need to consider the partner of the electron in the weak interaction: the neutrino, next section, before analyzing the electroweak theory within our formalism.

4.1. The Theory of the Neutrino

We develop here a theory of the neutrino which is the natural complement of the formulation we have shown above for the electron.

We mentioned that we can define the electron field in an operational form: it is that field Ψ_e which obeys the Dirac equation (and its gauging)

$$\begin{aligned}&(i\hbar D_0 - m)\Psi_e = 0,\\ &-(i\hbar D_0 - m)(-i\hbar D_0 - m) = -\hbar^2 D_0^2 - m^2\end{aligned} \tag{1}$$

then the field has the correct mass, charge and spin density (of course magnetic and electric moment when gauged by the electromagnetic field).

The neutrino is considered to be massless, spin $\frac{1}{2}$, and uncharged, then without magnetic or electric moment and allows no gauging by the electromagnetic field. We propose then an operational definition: the neutrino field corresponds to that field which obeys the equation

$$i\hbar D_n \Psi_\nu = 0, \quad D_n D_n^\kappa = \partial^\mu\partial_\mu \tag{2}$$

with D_n such that the neutrino, besides being massless ($m = 0$ in (2)) is also neutral, no electromagnetic gauging (coupling) allowed and has the correct spin and chirality (left handed).

This properties are obtained from the definitions

$$D_n = \gamma^0\partial_0 + i\gamma^5\gamma^j\partial_j, \quad j = 1,2,3, \\ D_n^\kappa = \gamma^0\partial_0 - i\gamma^5\gamma^j\partial_j \tag{3}$$

where we should remark that from the metric and the anticommuting properties of the basis vectors

$$\gamma^0\partial_0\gamma^0\partial_0 = \partial^0\partial_0, \\ -i\gamma^5\gamma^i\partial_i i\gamma^5\gamma^j\partial_j = -\partial^j\partial_j, \tag{4}$$

$$i\gamma^5\gamma^0\partial_0\gamma^j\partial_j - \gamma^j\partial_j i\gamma^5\gamma^0\partial_0 \\ = i\gamma^5\gamma^0\gamma^j(\partial_0\partial_j - \partial_j\partial_0) = 0. \tag{5}$$

Also, if a mass term $m \neq 0$ were included in (2), the spurious term $D_n m - m D_n^\kappa \neq 0$ would prevent the use of $m \neq 0$.

Additionally from (2) and (3) the requirement $i\gamma^5\Psi_\nu = -\Psi_\nu$ for the neutrino imposes the condition for it to be a left handed carrier field. (D_n^κ would be the operator for the right handed antineutrino field).

With respect to the gauging of the wave function Ψ_ν and the operator D_n, it is immediate that a term $\gamma^0 q_n A_0 g$ can not be cancelled by a gauge factor $g = e^{ia(t)/\hbar}$ acted upon by $i\hbar i\gamma^5\gamma^0\partial_0$ as far as $-\hbar\gamma^5\gamma^0\partial_0 g = -a'(t)\gamma^5\gamma^0 g$.

On the other hand a term $\gamma^5\gamma^0 A_0^{\text{axial}} g^{\text{axial}}$ will be cancelled by such a term. As in the case of the electron fields both possibilities are open, an axial electron current and an axial neutrino current can interact, this being the origin in our theory of the electron of the possible full electroweak interaction of the electron, but for the neutrino the interaction is restricted to the weak part (axial current only) and for the dual of the electron charge, that is g_D being the proper coupling constant.

4.2. The Electroweak Interaction of the Electron and the Neutrino

Now, from the description of the neutrino as above, and the possibility of both the electron left and right handed fields to be gauged by a pseudo-scalar-valued phase, a Lagrangian can be written in which both fields are together, the gauging of one corresponds to the opposite gauging of the other:

$$\nu_e + W^- \to e^-, \quad \text{or} \quad e^- + W^+ \to \nu_e \tag{6}$$

with the set of new gauge fields $W^-, W^+, [W^-, W^+] = Z^0$ carrying four physical properties: charge; angular momentum; vector ($SU(2)$) structure; and the possibility of interacting with carriers possessing a weak charge. Because this field interacts only with the left handed neutrino and the left handed part of the electron field, or the right handed anti-neutrino and the right handed part of the positron field, the weak field itself will have to interact with the mass producing phase factor of the LK theorem, then it will acquire mass from the same mechanism as the electron field (these matters are presented and analyzed in [1]-[4]).

4.3. Appendix. Chiral Symmetry in Complex Space–Time

We assume in accordance with the previous section that a local observer describes space–time by an orthonormal tetrad:

a) $(\gamma^0)^2 = -(\gamma^1)^2 = -(\gamma^2)^2 = -(\gamma^3)^2 = 1$; and $\gamma^\mu\gamma^\nu = -\gamma^\nu\gamma^\mu$ all $\mu \neq \nu$.

b) In this frame $\gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3$ is both the duality transformation operator and the pseudo-scalar $(\gamma^5)^2 = -1$.

It is important that another observer can use a different coordinate system related by a Lorentz transformation L, where the fundamental properties $(i\gamma^5)^2 = 1$ and $\gamma^5\gamma^\mu = -\gamma^\mu\gamma^5$ are also preserved, together with a) and b).

The handedness operator $H = i\gamma^5$ can be used to construct the chirality projectors P_R and P_L:

$$P_R + P_L = 1, \quad P_R P_R = P_R, \\ P_L P_L = P_L, \quad P_R P_L = P_L P_R = 0$$

where $P_R = \frac{1}{2}(1 + i\gamma^5)$, $P_L = \frac{1}{2}(1 - i\gamma^5)$ or $P_{R,L} = \frac{1}{2}(1 + \pm H)$.

If a coordinate transformation $\gamma^5 \to (\gamma^5)'$ is allowed where a), and consequently b), is not preserved (that is, if the determinant ζ of the transformation is not $\zeta = +1$) then $H \neq i(\gamma^5)'$ showing that a chirality operator $H = i(\gamma^5)'/\zeta$, with $H^2 = 1$ in all frames, has to be used. Here we shall assume $H = i\gamma^5$, because of the restriction a) and the assumption that we have selected a 'right' handed frame of reference. The P_R and P_L can better be considered numbers of a new mathematical field, complex space–time, with basis 1 and H, in an hyper-complexification of the Clifford algebra. H is coordinate invariant.

4.4. Appendix. The Weak Interaction

The weak interaction carries $W^\pm(\pm 2, \pm 1, \pm 1, \pm 1)$ information in relation to the t_μ numbers, with $SU(2)$ symmetry.

The color interaction carries $G_{cc'}(0, \pm 1, \pm 1, \pm 1)$ information in relation to the t_μ numbers, with $SU(3)$ symmetry.

An example of carrier interactions would be

$$\nu^0 + W^- \to e^-, \quad \text{or} \quad (-1,000) + (2,1,1,1) \to (1,1,1,1)\,,$$

for leptons and

$$u_r^{2/3} + G_{rb} \to u_b^{2/3}, \quad \text{or} \quad (-1,-1,0,-1) + (0,1,-1,1) \to (-1,0,-1,0),$$

for quarks.

We have shown [3] that they constitute a set with all the known properties of an elementary particle's family, the fields they represent can be:

— massless or massive after interactions are considered;

— charged (integer or fractional);

and (as is discussed in [1]) the collection of the fields constructed with (21) have weak charge and color, and in general the characteristics, usually postulated on a phenomenological basis, such as composites being colorless, confinement, etc. being immediate consequences of the defining equations.

The principle change from the usual presentation of the Standard Model is that now the equations have, as constitutional parts, a series of conditions corresponding to those which the phenomenological approach shows to be necessary. The conditions are here related to the basic properties of space–time as a frame of reference to describe physics.

Because of the appearance, or not, of the $i\gamma^5$ factors in (22), the fields have definite chiral properties. Only one field in the theory may have both chiralities simultaneously, and therefore can be, as a free field: massive, charged (reference charge ± 1), and weak charged. This is identified as the electron field. The different values of the index f, the family number, generate the families.

The resulting theory is a chiral geometry theory of charge, isospin and color. The theory has a Lagrangian formulation which reproduces all aspects of the standard theory. Even if the Higgs mechanism has, in its first approximation, the same motivation as in the standard theory, it has a purely geometric character in the present analysis.

The Lagrangian in the Standard Model for a fermion field with electroweak interactions and a symmetry breaking mass term is reproduced from the considerations above $\left(L_0 = \psi^{(0)}_{e_L},\ R_0 = \psi^{(0)}_{e_R}\right)$

$$\begin{aligned}\mathbf{L} = &+\frac{1}{2}\overline{L_0} i\gamma^\mu \\ &\times \left(\frac{1-i\gamma_5}{2}\right)\left(\partial_\mu L_0 + \frac{ig'}{2}B_\mu L_0 - \frac{ig}{2}A^i_\mu \tau_i L_0\right) \\ &- \frac{1}{2}\left(\partial_\mu \overline{L_0} - \frac{ig'}{2}B_\mu \overline{L_0} + \frac{ig}{2}\overline{L_0}\tau_i A^i_\mu\right) \\ &\times i\gamma^\mu\left(\frac{1-i\gamma_5}{2}\right)L_0 \\ &+\frac{1}{2}\overline{R_0} i\gamma^\mu\left(\frac{1-i\gamma_5}{2}\right)(\partial_\mu R_0 + ig' B_\mu R_0) \\ &-\frac{1}{2}(\partial_\mu \overline{R_0} ig' B_\mu \overline{R_0}) i\gamma^\mu\left(\frac{1-i\gamma_5}{2}\right)R_0 \\ &- g_e\left[\overline{R_0}\Phi^\dagger\left(\frac{1-i\gamma_5}{2}\right)L_0 \right. \\ &\left. + \overline{L_0}\left(\frac{1-i\gamma_5}{2}\right)\Phi R_0\right] \end{aligned} \tag{7}$$

where the $\tau_i (i = 1,2,3)$ are Pauli matrices, Φ is a field corresponding to the LK Theorem [4] and B_μ and A^i_μ are $U(1)$ and $SU(2)$ gauge fields. A further analysis of the fields in (7) shows that the coupling constants g and g' correspond to the electromagnetic constant e and to its dual (axial) pair.

To (7) we should add the energy corresponding to the neutrino, the energy corresponding to the interactions fields and the possibility of the neutrino and the electron interacting via the axial current, which by definition is also the basic current of the neutrino from its chiral properties as a massless field. The neutrino, as mentioned above, can not interact with its polar current without violating space–time symmetry.

Confinement results, within the theory, from the requirement that the Lorentz symmetry should not be broken even at local level. The same requirement gives rise to the colorless condition for hadrons, the new feature is that hadrons should be both globally and locally colorless. Fractional charges are also a natural consequence of the gauging properties of the Lagrangian.

The theory shows the reason for chirality being a basic property of nature as shown by the set of elementary carriers. This can be clearly seen with the gauging of the Diracon equations, following the discussion for the electron,

$$D_{(d,f)} = \Gamma^\mu_{(f)}\left[\partial^{(d)}_\mu - i\frac{e}{\hbar}A^{(d)}_\mu(x)\right] \tag{8}$$

the gauging fields having, by selection in agreement with $\partial_\mu^{(d)}$, the multi-vector composition

$$A_\mu^{(d)}(x) = A_\mu^{d,\text{scalar(electromagnetic)}} + A_\mu^{d,\text{pseudoscalar(weak,color)}} i\gamma^5 + A_{\alpha\beta,\mu}^{\text{tensor(gravity)}} \gamma^{\alpha\beta} \quad (9)$$

that is, the combined gauging has 1 + 2 + 3 + 6 parts given, when interacting with other carrier fields, electromagnetic, weak, color, and gravity parts. The first two terms carry the index (d) because they are relative properties. Then the wave function becomes upon gauging (φ a reference spinor).

$$\psi_d(x) = B \exp\{i(p_d^\mu x_\mu + \phi_d(x))\}\varphi \quad (10)$$

with the phase factor being a multi-vector

$$\phi_d(x) = \phi_{d,\text{scalar}}(x) 1 + \phi_{d,\text{pseudoscalar}}(x) i\gamma^5 + \phi_{d,\alpha\beta}(x)\gamma^{\alpha\beta} \quad (11)$$

the particular, relative, combinations for the phase, the $i\gamma^5$ terms, generate isospin and color and the $\gamma^{\alpha\beta}$ generate the local Lorentz transformations which are a consequence of gravity. The symmetries of

$$\phi_{d,\text{scalar}}(x) + \phi_{d,\text{pseudoscalar}}(x) i\gamma^5$$

generate the well known $SU(3)_c \otimes [SU(2) \otimes U(1)]_{ew}$ Standard Model.

The electroweak–color relative properties can be illustrated comparing the following complex currents vector for massless fields obeying (8). For a massless field $(j^0)^2 = (j^f)^2$ flowing, in the 8 cases below, in the $\mathbf{v}_f$ direction

$$\begin{aligned}
e\mathbf{j}_1 &= j^0 e_0 + \frac{1}{3} j^f (e_1 + e_2 + e_3), \\
d_r\mathbf{j}_2 &= j^0 e_0 + \frac{1}{3} j^f (e_4 e_1 + e_2 + e_3), \\
d_b\mathbf{j}_3 &= j^0 e_0 + \frac{1}{3} j^f (e_1 + e_4 e_2 + e_3), \\
d_g\mathbf{j}_4 &= j^0 e_0 + \frac{1}{3} j^f (e_1 + e_2 + e_4 e_3), \\
u_r\mathbf{j}_5 &= j^0 e_0 + \frac{1}{3} j^f (e_1 + e_4 e_2 + e_4 e_3), \\
u_b\mathbf{j}_6 &= j^0 e_0 + \frac{1}{3} j^f (e_4 e_1 + e_2 + e_4 e_3), \\
u_g\mathbf{j}_7 &= j^0 e_0 + \frac{1}{3} j^f (e_4 e_1 + e_4 e_2 + e_3), \\
\nu\mathbf{j}_8 &= j^0 e_0 + \frac{1}{3} j^f (e_4 e_1 + e_4 e_2 + e_4 e_3),
\end{aligned} \quad (12)$$

from the property $(e_4)^2 = 1$ we see that if we make the replacement $j^f \to j^f e_4$ the current $\mathbf{j}_8$ will be transformed into current $\mathbf{j}_1$ and the currents $\mathbf{j}_{5,6,7}$ into the currents $\mathbf{j}_{2,3,4}$ respectively, corresponding to the $SU(2)$ symmetry and an equivalent type of transformation, for the sets $\{2,3,4\}$ and $\{5,6,7\}$ within themselves, corresponding to the $SU(3)_c$ symmetry.

The semi-empirical mass matrix (based on Królikowski [7]) for the families of elementary carriers has a very interesting form in its first approximation:

$$m_{(f,d)} = N_f m_d (5.75(1 - (1-f)K_f/c_f^2) + \text{effect of nondiagonal terms}) \quad (13)$$

with $N_f = n_f c_f^2$ and $m_d = m_0 (n_c)_d^2 Q_d^4$, where n_f is the degeneracy of the family's wave function as mentioned above, $c_f = 2f - 1$ the number of spinors in the outer product of ψ, m_0 the electron mass, n_c the number of color degrees of freedom: 1 for ν, e^- and 3 for the quarks; and Q_d the charge of the lepton or quark field. Then the masses are all, in a first approximation, proportional to the electron mass. That is, the action density is multiplied by this factor of geometrical origin. The factor Q_d^4 suggests that the additional interaction is directly related to the electronic, gauge, field as of a self-interaction origin. The induced mass and charge discussions below are related to this point. The creation of a pair of elementary carriers at a given point, and its subsequent separation, involves the creation of their gauge fields. Q_d^2 is the factor for the energy required to create the carrier's electromagnetic field, an inseparable field from the concept of the existence of the carrier, whereas Q_d^4 should correspond to geometric self interaction.

In the theory we have presented here the physical properties are now a constitutive part of the wave equations.

No additional isospin space is therefore needed, it is generated by the relative properties of the fields, the same applies to the color space.

Nucleons like proton or neutron and mesons are, within this theory, composite fields but elementary carriers. In fact these composite 'elementary' particles cannot, even if enough energy is available, be split into smaller components; the requirement of rotational invariance forces the 'colorless' combination of quarks, even to the smallest possible experimental probe size.

Otherwise the use of STA and its equivalent complex space–time, which results in a five-dimensional geometry, with 32 geometric degrees of freedom, allows the construction of a theory with both induced matter and interaction fields and the new features consisting in a natural existence of the $SU(3) \otimes SU(2) \otimes U(1)$ theory for the elementary particles fields. The main difference is that here we do not have a model (corresponding to the Standard Model) but a theory of the elementary carriers and their interactions fields, where the SM is a natural structural part.

Acknowledgement

The author is a member of the Sistema Nacional de Investigadores, Conacyt, México. The technical assistance of Mrs. Irma Vigil de Aragón is gratefully acknowledged.

References

[1] Keller, J.: Advances in Applied Clifford Algebras. **9** (2), 309–395 (1999).

[2] Keller, J.: Rev. Soc. Quim. Mex. **44**(1), 22–28 (2000).

[3] Keller, J.: The Theory of the Electron; A Theory of Matter from START. Foundations of Physics Series 117. Dordrecht: Kluwer Academic Publishers (2001).

[4] Keller, J.: Unification of Electrodynamics and Gravity from START, Annales de la Fondation Louis de Broglie.**27**(S) 359-410(2002); Keller, J.: Advances in Applied Clifford Algebras. **11** (S2), 183-204 (2001)

[5] Keller, J. and Weiberger, P.: A Formal definition of Carriers, Advances in Applied Clifford Algebras. **12** (1), 39–62 (2002).

[6] Keller, J., Keller, A., Flores, J.A.: Acta Chimica Theoretica Latina **XVIII** (4), 175–186 (1990)

[7] Królikowski, W.: Acta Physica Polonica. B21: 871–879 (1990); Phys. Rev. D 45: 3222 (1992).

[8] Halsen, F. and Martin, A.D.: Quarks and Leptons. New York: Wiley (1984).

[9] Charlier, A., Bérard, A., Charlier, M-F. and Fristot, D. Tensor and the Clifford Algebra. New York:Marcel Dekker, Inc. (1992).

[10] de Broglie, L.:Annales de Physique, **3**, 22, (1925).

[11] Schrödinger, E.: Annalen der Physik, (**79**, p.361, 489, 734; **81**, p.109) (1926)

[12] Snigg, J.: Clifford Algebra. A computational Tool for Physicists. New York:Oxford University Press (1997).

[13] Spencer, A.J.M.: Continuum mechanics. New York: Longman (1980).