PACS №: 11.30.Rd; 03.65.Pm

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Massive Particles with Definite Chiralities

Contents

Abstract

An antilinearly modified Dirac equation with two mass parameters is introduced. This equation is solved by two independent states of definite (and opposite) chiralities, each state being either massive or massless. Possible applications in neutrino physics are pointed out.

1. Introduction

In previous papers [1–6], the Dirac equation with two mass parameters and related topics were discussed. The approach was used to derive standard equations for massive, massless and tachyonic fermions. In particular, a massless equation was obtained, which differs from the usual one and does not produce a superfluous conserved current. In Ref. [7], the aforementioned results were reformulated and justified on the grounds of desirable features relating to the active symmetry operations (time reversal, spatial parity, etc.). Possible applications and a flavored neutrino model were examined in Refs. [1, 3, 7] and [8]. The present paper introduces an antilinear modification and generates a formalism of two independent chiral eigenstates, each being either massive or massless. This description is quite different from that of Refs. [1–8], but, again, could be useful for neutrino physics, as pointed out in the Conclusions. For related approaches, see: Refs. [9–12].

The treatment is done before second quantization, and notation is rather conventional. Specifically, and unless otherwise noted, Greek (Latin) indices run through the values $0, 1, 2, 3$ $(1, 2, 3)$ and the summation convention is applied to repeated up and down labels. Units are such that $\hbar = c = 1$. An attempt is made at distinguishing powers from superscripts: for instance, $(\not{P})^2$ and $|a|^2$ are powers, while γ^0 indicates a specific object with superscript 0. The curly bracket is used for ordered sets: e.g., $\{x^\lambda\}$ denotes four objects in the order 0–3.

2. Dirac Equation with Two Mass Parameters

In a frame of reference $\mathcal{X}$ of real spacetime coordinates $x = \{x^\lambda\}$ and pseudoeuclidean metric $g^{\mu\nu} = \mathrm{diag}\{+1, -1, -1, -1\}$, the Dirac equation with two mass parameters [1] may be written as follows

$$\not{P}\Psi(x) = \mathrm{M}\Psi(x) \tag{1}$$

with

$$\not{P} = \mathrm{i}\gamma^\alpha \partial_\alpha \tag{2}$$

and

$$\mathrm{M} = a\mathrm{M}^{(-)} + b\mathrm{M}^{(+)} \tag{3}$$

where a and b are complex constants, $\Psi(x)$ is a complex four-spinor, and $\mathrm{M}^{(\mp)}$ indicate the chiral projectors:

$$\mathrm{M}^{(\mp)} = \frac{1}{2}\,(I \mp \varepsilon\gamma^5). \tag{4}$$

The Dirac matrices γ^λ (in a fixed chosen representation) obey the usual rules

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}I, \qquad (\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0 \tag{5}$$

with I being the 4×4 identity matrix. The matrix $\gamma^5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3$ is hermitian and unitary, and anticommutes with all γ^λ. For general reference on the Dirac equation and related topics, see, for instance: Refs. [13–23] The value of the sign $\varepsilon = (-1)^{T+S}$ depends on the frame of reference [1, 24]. Namely, the time-index T and the space-index S of $\mathcal{X}$ are so defined: $T = 0$ if $t = x^0$ runs forward ($T = 1$ otherwise) and $S = 0$ if $s = \{x^\ell\}$ is a right-handed triplet ($S = 1$ otherwise). It is also reminded:

$$(\gamma^\mu)^* = \mathrm{B}^\dagger\gamma^\mu\mathrm{B}, \qquad (\gamma^5)^* = -\mathrm{B}^\dagger\gamma^5\mathrm{B} \tag{6}$$

where B is the (fixed chosen) unitary matrix associated with the charge conjugation operation [2], and the asterisk denotes complex conjugation.

The solutions $\Psi(x)$ of Eq. (1) are eigenstates of the squared four-momentum operator [1]

$$\Box = (\not{\mathcal{P}})^2 = -\partial_\alpha g^{\alpha\beta}\partial_\beta \tag{7}$$

for the eigenvalue ab. Six cases can be identified:

(I) $a = 0 = b$;

(II) $a \neq 0, b = 0$;

(III) $a = 0, b \neq 0$;

(IV) $ab > 0$;

(V) $ab < 0$;

(VI) $ab \notin \mathrm{Re}$.

Case (VI) is of unclear interpretation, and the other cases were studied in Ref. [7].

3. Antilinear Modification

Equation (1) admits an interesting modification of the following type:

$$\not{\mathcal{P}}\Psi(x) = \mathrm{M}[\Psi(x)]^{\mathrm{C}}. \tag{8}$$

The symbol C denotes the antilinear operation of charge conjugation [2], defined as

$$[\Phi(x)]^{\mathrm{C}} = \gamma^5\mathrm{B}\Phi^*(x) \tag{9}$$

on a generic four-spinor $\Phi(x)$; the notation $\Phi^{\mathrm{C}}(x)$ will also be used when convenient. Equation (8) is manifestly covariant under changes of coordinates of the Poincaré group, provided a and b are treated as scalars, and the usual (passive) spinor transformations [19] are adopted with an appropriate phase convention: i.e., see Eq. (37) of Ref. [2].

For the active operations [2] of charge conjugation, spatial parity (P), time reversal (T), PC and TPC

$$\begin{aligned} \Phi^{\mathrm{P}}(x) &= \mathrm{i}\gamma^0\Phi(t, -s), \\ \Phi^{\mathrm{T}}(x) &= \gamma^0\mathrm{B}\Phi^*(-t, s), \end{aligned} \tag{10}$$

$$\begin{aligned} \Phi^{\mathrm{PC}}(x) &= \mathrm{i}\gamma^0\gamma^5\mathrm{B}\Phi^*(t, -s), \\ \Phi^{\mathrm{TPC}}(x) &= -\mathrm{i}\gamma^5\Phi(-x), \end{aligned} \tag{11}$$

one obtains that:

(i) TPC invariance is valid for all possible choices of a and b;

(ii) invariance under C applies if $a = b^*$;

(iii) invariance under P is valid if $a = b$;

(iv) invariance under T applies if $a, b \in \mathrm{Re}$;

(v) invariance under PC is valid if $a, b \in \mathrm{Re}$.

For example, the equations for $\Psi^{\mathrm{C}}(x)$ and $\Psi^{\mathrm{T}}(x)$ are as follows:

$$\not{\mathcal{P}}\Psi^{\mathrm{C}}(x) = \Big(b^*\mathrm{M}^{(-)} + a^*\mathrm{M}^{(+)}\Big)\,[\Psi^{\mathrm{C}}(x)]^{\mathrm{C}}, \tag{12}$$

$$\not{\mathcal{P}}\Psi^{\mathrm{T}}(x) = \Big(a^*\mathrm{M}^{(-)} + b^*\mathrm{M}^{(+)}\Big)\,[\Psi^{\mathrm{T}}(x)]^{\mathrm{C}}. \tag{13}$$

In closing this section, note that Eq. (8) leads to the generalized (linear) Klein-Gordon equation

$$\Box\Psi(x) = \mathrm{N}\Psi(x), \tag{14}$$

with

$$\mathrm{N} = |b|^2\mathrm{M}^{(-)} + |a|^2\mathrm{M}^{(+)}. \tag{15}$$

Furthermore, observe that Eq. (8) is nearly linear, but not exactly linear: specifically, if $\Psi_1(x)$ and $\Psi_2(x)$ are solutions of (8), their linear combination may not be a solution of (8) unless the coefficients of the combination are real. The physical meaning of this feature may be explored in future work; also, a possible Lagrangian formalism will be investigated.

4. Chiral Components

The definitions

$$L(x) = \mathrm{M}^{(-)}\Psi(x) \qquad R(x) = \mathrm{M}^{(+)}\Psi(x), \tag{16}$$

split Eqs. (8) and (14) into left-handed and right-handed equations:

$$\not{\mathcal{P}}L(x) = b[L(x)]^{\mathrm{C}}, \qquad \Box L(x) = |b|^2L(x), \tag{17}$$

$$\not{\mathcal{P}}R(x) = a[R(x)]^{\mathrm{C}}, \qquad \Box R(x) = |a|^2R(x) \tag{18}$$

with separately conserved real currents

$$\begin{aligned} j^\mu(x) &= L^\dagger(x)\gamma^0\gamma^\mu L(x), \\ k^\mu(x) &= R^\dagger(x)\gamma^0\gamma^\mu R(x), \end{aligned} \tag{19}$$

each current being appropriate for the usual probability interpretation in terms of a single particle

"Electromagnetic Phenomena", V.3, №1 (9), 2003 **47**

theory [7, 19]. Thus, Eq. (8) describes two Dirac particle states, each with a definite chirality. If $b \neq 0$ ($a \neq 0$) the spinor L (R) is massive; otherwise, it is massless. Tachyonic cases are not allowed.

5. Conclusions

An interesting formulation consists in taking $b = 0$ and $a \neq 0$, so that one has the (linear) massless equations

$$\not{P}L(x) = 0, \qquad \Box L(x) = 0 \tag{20}$$

with current

$$j^\mu(x) = L^\dagger(x)\gamma^0\gamma^\mu L(x) \tag{21}$$

and the massive equations

$$\begin{aligned} \not{P}R(x) &= a[R(x)]^{\mathrm{C}}, \\ \Box R(x) &= |a|^2 R(x), \end{aligned} \tag{22}$$

$$k^\mu(x) = R^\dagger(x)\gamma^0\gamma^\mu R(x)\,. \tag{23}$$

Equations (20),(21) display the same continuous global phase invariance of the standard massive Dirac equation [2, 19]: thus, this invariance can be gauged (i.e., made local) by means of the minimal replacement, in order to provide interaction with a vector field. On the other hand, Eqs. (22),(23) do not share this property, and a vector interaction may not be warranted: the particle remains “sterile”. The model may find application in the description of a massless, interacting, left-handed neutrino, possessing a massive, chirally right-handed, sterile counterpart. Each neutrino is distinct from its antiparticle. Presumably, the sterile neutrino would still interact gravitationally with a mass $\mid a \mid$, and be part of the dark matter [25] of the universe. In second quantization, it appears that the massive current does not remain conserved if quantized with anticommutators.

Acknowledgements

The author wishes to thank Prof. Valeri V. Dvoeglazov (Escuela de Fisica, Universidad Autonoma de Zacatecas, Mexico) for useful discussions.

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